Coexisting Exchange Platforms: Limit Order Books and Automated Market Makers

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Abstract

Blockchain-based decentralized exchanges have adopted automated market makers—algorithms that pool liquidity and make it available to liquidity takers by automatically determining prices. We develop a theoretical framework to analyze coexisting market-making structures: a traditional centralized limit-order market and a decentralized automated market. Traders face asymmetric information and endogenously choose trading venues. We show that liquidity on the automated market complements that on the limit-order market. A unique and stable general equilibrium exists with endogenous liquidity on both platforms, and we investigate the impact of adopting automated market makers on asset prices and traders' behavior.

Keywords: decentralized exchanges, automated market makers, limit order books, liquidity, asymmetric information

JEL code: G1, D4

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1 Introduction

Limit order books are a core trading mechanism in the modern electronic financial market. Traders called market makers provide trading opportunities by placing *limit orders* and by quoting prices at which they are willing to buy or sell a certain amount of an asset. Limit orders are stored in a limit order book (LOB) and publicly displayed. Liquidity takers then place marketable limit orders or *market orders*.[1] An incoming market order is matched with standing limit orders on the book and is executed at the proposed bid or ask price.

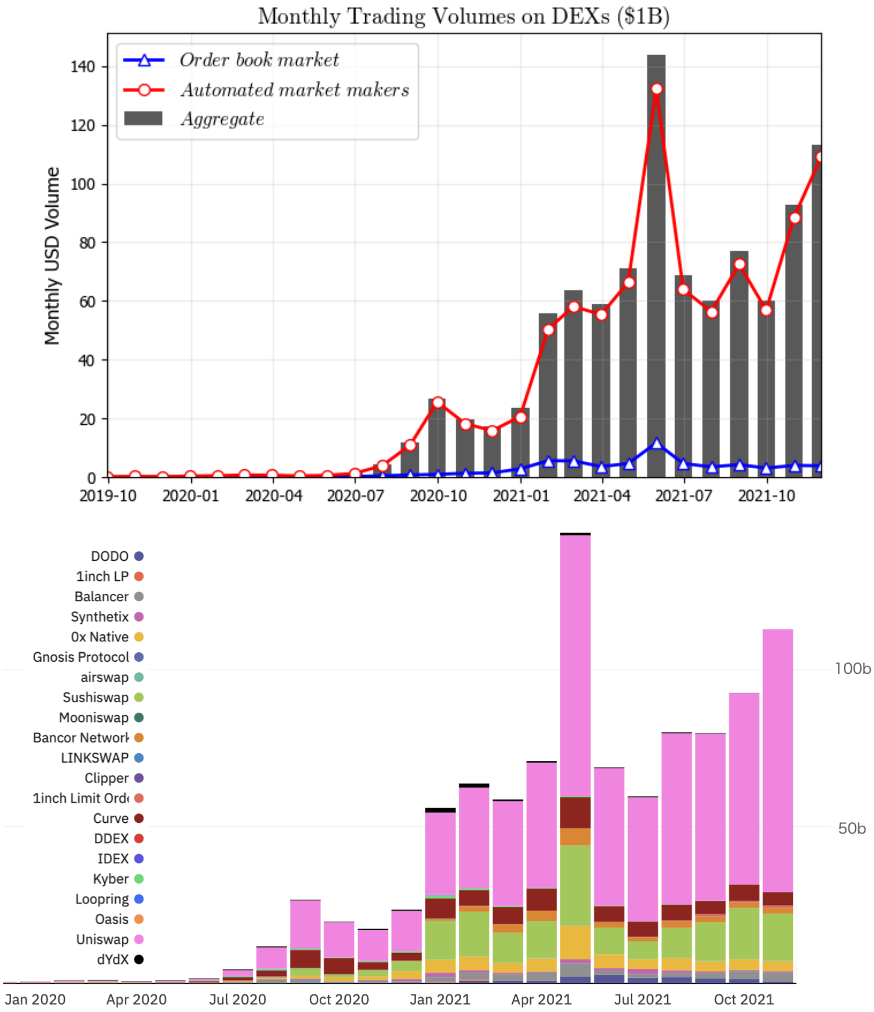
The recent upsurge in cryptocurrency and blockchain, however, has changed the landscape of market structures. Many exchange platforms are built on smart contracts on the Ethereum blockchain, and transactions are executed in a decentralized manner. These platforms are called decentralized exchanges (DEXs) in comparison with traditional centralized exchanges (CEXs). They have attracted a sizable trading share in transactions involving digital assets, as the upper panel of Figure 1 illustrates.[2] Moreover, DEXs have introduced pricing and matching algorithms called *automated market makers* (AMMs), and they play a substantial role in the prosperity of DEXs.[3] As a result, two different market-making algorithms coexist in the markets for digital assets: traditional limit order books and AMMs. We propose a theoretical framework to analyze this situation.

[1]See, for example, Li, Ye and Zheng (2021) for discussions of other sophisticated order types.

[2]Traditional CEXs (such as Bittrex and Binance) are characterized by a centralized authority who manages trader funds, requires KYC information, and controls trade-related functionalities of an exchange. In contrast, DEXs are built on the blockchain with decentralized information management systems. Different categories of DEXs exist (see, for example, Totle, 2019), and this paper focuses on "on-chain" DEXs, in which all but infrastructure and development are decentralized.

[3]The idea of AMMs was previously proposed and implemented in the context of prediction markets. However, before the advent of exchanges for digital assets, AMMs were not adopted in markets with real money, such as equity markets. See, for example, Hanson (2003), Chen, Fortnow, Lambert, Pennock and Wortman (2008), and Abernethy, Chen and Wortman Vaughan (2011).

Figure 1: Monthly trading volume on DEXs on Ethereum



Note: The top panel plots the monthly trading volume on DEXs denominated by USD. The bottom panel plots the monthly trading volume by DeFi projects. They include all DEXs on the Ethereum blockchain through December 2021. Source: Dune Analytics (duneanalytics.com)

In contrast to a limit-order market, where participants trade with each other, in an automated market, participants trade against *liquidity pools*, i.e., pools of assets reserved on an exchange. AMMs determine asset prices (or exchange rates) following a pre-determined algorithm by taking the state of the pools as an input. AMMs do not require the physical presence of active market makers or dealers for pricing and order execution and consume much less memory than the traditional order-book algorithm, allowing a substantial proportion of trades to be on the blockchain.[4]

AMMs are classified into several types according to the pricing function. The lower panel of Figure 1 shows that constant product market makers (CPMMs), adopted by Uniswap, Sushiswap, and PancakeSwap, are a dominant market structure.[5] In addition, Balancer has attracted traders by adopting constant mean market makers (CMMMs), while Curve has introduced a hybrid function. In this paper, we analyze a general form of AMMs called constant function market makers (CFMMs). CFMMs nest most of the real-world implementations, such as CPMMs, CMMMs, and hybrid functions, and provide implications with a broader generality.[6]

AMMs work as follows (Subsection 2.2 provides more details). Liquidity providers lock traded assets into the exchange, and AMMs aggregate them to create liquidity pools. Suppose that the liquidity pools reserve x and y units of token X and token Y prior to a trade. If a trader buys δ units of token Y by paying $P\delta$ of token X, she moves the liquidity pools from (x, y) to $(x', y') = (x + P\delta, y - \delta)$. For example, the CPMM algorithm requires the (squared) geometric mean of the liquidity pools to be constant, $xy = x'y'$. This single equation derives the price of token Y in terms of X as $P = \frac{x}{y-\delta}$. In general, CFMMs are characterized by a certain function f and impose condition $f(x', y') = f(x, y)$ to derive a price as a function of the trading volume (δ) and the amount of liquidity supply (x, y). Liquidity in an automated market is measured by

[4]Harvey, Ramachandran and Santoro (2021) propose five problems in traditional centralized finance (CeFi) that decentralized finance (DeFi) may solve: inefficiency, limited access, opacity, centralized control, and interoperability. In this paper, we take these motivations to adopt DeFi as given and focus on the economic consequences of the adoption of DeFi.

[5]PancakeSwap is not on the figure because it operates using Binance Smart Chain.

[6]Few exceptions include constant sum market makers. Also, Uniswap v3 has implemented a more complicated pricing function than CPMMs by allowing liquidity providers to supply liquidity within a certain range of prices. Our results regarding traders' platform choice are robust to Uniswap v3 as long as the bonding curve satisfies the regularity conditions in Appendix C.

the amount of assets locked in the platform, i.e., x and y. The incentive for liquidity providers stems from fluctuations in the pools' value caused by a trade (i.e., $x' - x$ and $y' - y$), as they withdraw and liquidate their contribution when they exit the market.[7]

This paper studies how the introduction of CFMMs affects the liquidity of the entire market when traders face an asymmetric information problem. Importantly, our model features coexisting exchanges with two different market-making algorithms: a DEX with a CFMM and a CEX with a limit order book. There are informed for-profit traders, uninformed liquidity (noise) traders, and market makers, and they endogenously choose their trading platforms. We first analyze the consequences of an exogenous variation in DEX liquidity for traders' behavior and its impact on CEX liquidity. We then endogenize liquidity provision by market makers on the DEX and describe how liquidity on the DEX and the CEX jointly reacts to greater informational friction.

Liquidity on the DEX complements that on the CEX. This is because informed traders and liquidity traders exhibit disproportional reactions to increases in the DEX liquidity pools. Larger liquidity pools mitigate the price impact of a liquidity-taking order, and informed traders enjoy this effect because they tend to cluster on the same side of the DEX, incurring a large price impact. Thus, larger liquidity pools attract more informed traders to the DEX. In contrast, the reaction of liquidity traders to additional liquidity tends to be weak. Their trading behavior stems from random exogenous reasons, such as margin calls or hedging needs, and random buy and sell orders tend to cancel each other out on the DEX. This leads to a small expected price impact, and deeper liquidity on the DEX has a limited effect on liquidity trading. As a result, more informed traders than liquidity traders migrate from the CEX to the DEX, leading to a less severe adverse selection problem and deeper liquidity on the CEX.

We then formulate the expected profit function for liquidity providers on the DEX with asymmetric information. As the existing theories suggest (e.g., Angeris and Chi-

[7] In reality, liquidity providers also obtain fees, staking rewards, and governance tokens, but we abstract away from these exogenous factors in this paper. See Subsection 6.2 for more details.

tra, 2020), liquidity providers suffer from a cost called *impermanent loss*. It emanates from informed traders imposing an adverse selection cost on uninformed liquidity providers (Glosten and Milgrom, 1985). In contrast to the literature, however, liquidity providers in our model also gain lucrative trading opportunities, as the expected value of liquidity pools improves when a trade is initiated by an uninformed liquidity trader. The profit opportunity is hard-wired in the convexity of the CFMMs' pricing algorithm: when the liquidity pools randomly fluctuate along the convex curve, their expected value improves due to Jensen's inequality.[8] Therefore, liquidity providers endogenously determine their liquidity supply by weighing the impermanent loss against the profit from noise trading. As in limit-order markets, liquidity in an automated market is negatively affected by the signal-to-noise ratio of order flows.

Our model proposes several important empirical implications. Since CFMMs are a type of convex pricing, consuming liquidity involves a larger price impact than adding it. In other words, given the trading size, a buy order tends to be more costly than a sell order. Due to this asymmetric price impact on the DEX, bid and ask prices on the CEX also tend to be asymmetrically distributed around the expected value of the asset, leading to a biased midpoint quote compared to the expected asset value.

Also, buy and sell orders predict the future asset return with heterogeneous precision. The above-mentioned asymmetric price impact makes DEX buyers more reactive than sellers to exogenous variations in deep parameters. When the DEX exogenously becomes less attractive, for example, a sell order flow tends to be more informative and followed by a negative innovation in the asset's return compared to a buy order flow followed by a positive innovation. This is because informed buyers on the DEX are more likely to switch their trading venue than informed sellers, whereas liquidity

[8]The main model considers a one-shot trading game without fees for simplicity. Thus, liquidity providers withdraw assets immediately after a trade to avoid further impermanent loss caused by arbitragers. In Subsection 6.2, we discuss the case where liquidity providers earn fee revenues from transactions but must also pay blockchain mining fees to withdraw liquidity.

traders exhibit a relatively weak reaction compared to informed traders due to their limited expected price impact. These results are indicative of the market reaction to, for example, listings of new cryptocurrency/token pairs on a DEX, such as Wrapped Bitcoin (WBTC) and Wrapped Ethereum (WETH) on Uniswap.

The last part of the paper provides welfare analyses. Since trading is a zero-sum game, the aggregate welfare depends on the exogenous private utility of liquidity traders. In the model, we assume that they have heterogeneous preferences for the DEX or the CEX, e.g., aversion toward delays on the DEX and cyber security risks on the CEX. Thus, whether the DEX improves welfare crucially depends on the modeling assumption of liquidity (noise) traders. However, we show that the absolute welfare impact of the introduction of the DEX can be measured by observing the bid-ask spread on the CEX, as the spread determines the measure of liquidity traders who are willing to use the DEX.

Related literature. This paper is built on the large body of literature on market microstructure. In particular, Glosten and Milgrom (1985) and Kyle (1985) provide models of market liquidity with asymmetric information, following the conceptualization of Bagehot (1971). We apply their canonical framework to the new context of decentralized exchanges and show that adverse selection still plays a key role in explaining liquidity provision in a market with AMMs.

The modern financial market has experienced the fragmentation of trading exchanges, and several papers have addressed the implications of coexisting exchange platforms with different market microstructures, such as dark pools (Ye, 2011; Zhu, 2014; Ye, 2016), heterogeneous latency and transparency (Lee, 2019), and speed bumps (Brolley and Cimon, 2020). Our model also sheds light on the liquidity impact of heterogeneous market structures in the era of decentralization and blockchain.

Moreover, it contributes to a general understanding of blockchain, cryptocurrency, and decentralized exchanges. The literature is expanding (see Chen, Cong and Xiao, 2019, and Harvey, Ramachandran and Santoro, 2021 for comprehensive reviews), and many authors have analyzed the blockchain protocol as a new platform for value transfer, including Chiu and Koeppl (2017), Malinova and Park (2017), Pagnotta and Buraschi (2018), Schilling and Uhlig (2019), Abadi and Brunnermeier (2018), Cong, Li and Wang (2021), and Huberman, Leshno and Moallemi (2021). However, these studies either consider order book markets only or abstract away from the formal description of matching or pricing algorithms on decentralized platforms. We seek to provide further insights by incorporating AMMs into the equilibrium analyses as a dominant market-making algorithm on DEXs.

Although the research on AMMs is in its infancy, Angeris, Kao, Chiang, Noyes and Chitra (2019) provide a model of the optimal arbitrage problem with constant product market makers, and Angeris and Chitra (2020), Evans (2020), and Angeris, Evans and Chitra (2020) generalize analyses to the case with CFMMs.[9] Several complementary papers have also analyzed the market microstructure of AMMs. Lehar and Parlour (2021) compare the returns for liquidity providers on limit order markets and automated markets separately. Capponi and Jia (2021) develop a game theoretic model of liquidity provision via AMMs, investigating the possibility of a liquidity freeze and the impact of AMMs on other decentralized applications.[10] Park (2021) points out that AMMs harm efficiency because the algorithmic pricing facilitates economically meaningless transactions, such as front-running. Han, Huang and Zhong (2021) empirically attest that traders respond to prices both on CEXs and DEXs, rather than referring to the price on only one type of exchange. Our model complements the above studies by developing the first theoretical framework to analyze *coexisting* AMMs and limit-order

[9] Implementational details of decentralized exchanges are provided by, for example, Warren and Bandeali (2017), Zhang, Chen and Park (2018), and Adams, Zinsmeister and Robinson (2020).

[10] In contrast to our paper, Capponi and Jia (2021) do not rely on information asymmetry. In such a situation, they show that the liquidity providers' willingness to supply liquidity can be encouraged by a larger impermanent loss.

markets.

2 Technology Overview

We briefly describe trade execution on decentralized exchange platforms using a CPMM, a leading example of a CFMM (or an AMM in general). Appendix A provides an overview of the blockchain technology.[11]

2.1 Decentralized Exchanges

Building a trading platform on the blockchain (i.e., a decentralized exchange) is a natural strategy to extricate financial trading from a centralized information management and to make it robust to cyber attacks or single point of failures. As suggested by Harvey, Ramachandran and Santoro (2021), decentralization in finance is expected to improve traditional finance in many aspects, such as efficiency, transparency, and limited access. However, maintaining a limit order book by a smart contract on the Ethereum blockchain is costly and tends to be slow, due to the time-consuming mining process, a complicated matching mechanism of limit order books, and the limited capacity of the blockchain.

As a first solution, several DEXs have adopted "hybrid" mechanisms that involve both on-chain and off-chain features.[12] However, the hybrid system still uses centralized protocols to a certain extent. The second solution is AMMs. As mentioned in the introduction, an AMM is a pre-determined algorithm that sets a price for order execution. As it is simpler than a limit-order matching mechanism, it requires much less

[11] Readers can refer to Antonopoulos (2014) and Antonopoulos and Wood (2018) for more details.

[12] For example, `0x` is built on the so-called *relayer mechanism* (see Warren and Bandeali, 2017). It provides an off-chain order book, on which traders can broadcast their intentions and find their counterparties. Since the order book is maintained off-chain, it refreshes swiftly. Once traders agree on a transaction (i.e., trade execution), the order is settled on the blockchain via smart contracts.

computational capacity, making trade on the blockchain easier and faster.

2.2 Constant Product Market Makers

Liquidity pools and asset prices. Consider token X and token Y. Market makers inject tokens into an exchange following a certain rule described below. The exchange aggregates locked tokens and creates *liquidity pools*. Suppose that the exchange reserves x units of token X and y units of token Y. The CPMM requires the geometric mean of the liquidity pools to be constant. In particular, the initial liquidity pools define a constant, $k = xy$, and the prices for subsequent transactions are determined so that the product of liquidity pools stay the same at the initial level (before transaction fees are incorporated).

If a trader wants to buy Δx of token X by selling $\Delta y = P\Delta x$ of token Y at price P, she adds Δy of token Y to the pool and withdraws Δx of token X. This triggers the following change in the pools: $x \to x' = x - \Delta x$, and $y \to y' = y + \Delta y$. Since the geometric mean of the pool must be constant, the price must satisfy the following equation.

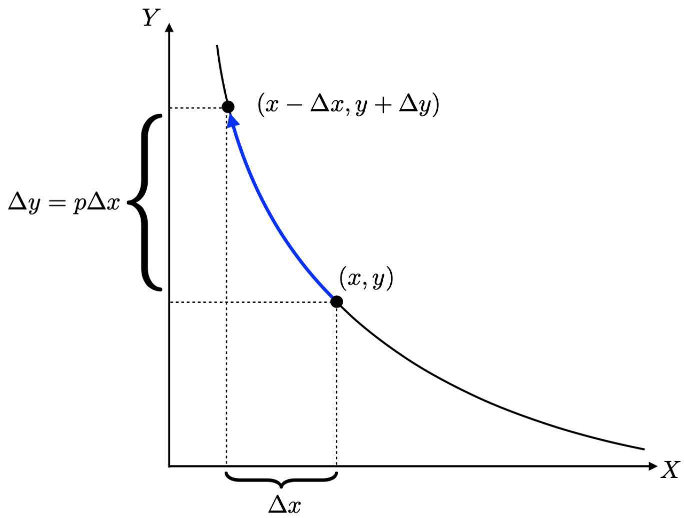
$$x'y' = (x - \Delta x)(y + P\Delta x).$$

The above equation determines P as a function of the current state of the pool, (x, y), and the trading quantity, Δx:

$$P = \frac{y}{x - \Delta x}.$$

The larger the quantity the trader wants to buy ($\Delta x > 0$), the higher the price she must pay, i.e., the price is an upward-sloping curve in the trading quantity.

Figure 2: Constant product market makers



Note: This figure illustrates a change in the state of liquidity pools when an incoming market order is buying Δx units of token X. The CPMM requires the liquidity pools to stay on the convex curve by adjusting for a change in token Y or, equivalently, the execution price p.

Also, considering a small trading volume, $\Delta x \to 0$, the execution price for an infinitesimal trade is given by $p \equiv \lim_{\Delta x \to 0} P(\Delta x) = y/x$, that is, the relative size of the liquidity pools. p is referred to as the *marginal* price of the asset when the liquidity pools have (x, y). Figure 2 shows a change in the pools' state caused by the above transaction: the marginal price is determined by the slope of the curve specified by $k = xy$.

Moreover, the pricing algorithm of the CPMM (or AMMs in general) satisfies the property called *path independence*, i.e., when the liquidity pools move from one state to another, the expected execution price is independent of the paths that the pools take. In the context of our paper, this means that trading a certain amount of assets all at once is equivalent to splitting orders and trading sequentially.[13] Appendix D provides a formal proof for this point.

[13] As discussed by Angeris et al. (2019), splitting orders costs more than trading all at once if a trader must pay a fee for trade execution.

Liquidity providers. When a market maker (or a liquidity provider) supplies liquidity via the CPMM, she is required to lock both token X and token Y. The amount of supplied liquidity must be adjusted so that the price of an infinitesimal trade does not change (see Subsection 3.3 for a more detailed analysis). A transaction triggers $(x, y) \to (x', y')$, and the change in the value is a source of their profit (or a cost). In particular, if market maker i injects (x_i, y_i) before a trade, and the aggregate size of the pools is (x, y), she obtains the share of the pools, $w = \frac{px_i + y_i}{px + y}$. Once a trade is executed, the market maker can withdraw her share from the post-trade liquidity pools, (x', y'), and realize her returns. On Uniswap, for example, it is free to withdraw liquidity at any time without lockup periods.[14]

Constant function market makers. Our main model considers a CFMM. This is a broader class of AMM characterized by a certain function $f : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ which maps the current state of the liquidity pools to some constant.[15] With the above example, the execution price P is determined by

$$f(x, y) = f(x - \Delta x, y + P\Delta x).$$

f must satisfy some regularity conditions so that $P \geq 0$ is uniquely determined by the above equation (see Subsection 3.3 for formal analyses). Note that the CPMMs are a special case of the CFMMs with $f(x, y) = xy$.

3 The Model

[14] See, for example, https://blog.orbsdefi.com/p/how-to-withdraw-liquidity-from-uniswap

[15] We assume that the automated market deals with exchanges of two assets but, in general, it can be defined with $n \geq 2$ assets by considering $f : \mathbb{R}^n_{++} \to \mathbb{R}_{++}$.

3.1 Environment

Consider a trading game in a two-period economy ($t = 0, 1$) with three types of traders: informed traders, liquidity traders, and market makers. They trade a single risky asset with an initial common value v_0.

Throughout the paper, we use the term *asset* to underscore the model's generality, and cash (e.g., USD) serves as the numeraire. Alternatively, we can think of the asset as a digital token (e.g., an ERC-20 token) and cash as another risky token or a stable coin, with v_0 being their relative value. With this interpretation, the asset price represents the exchange rate between tokens.

Events and traders. One of two possible event types triggers a trade at $t = 1$: either an innovation in the value of the asset (a common-value shock) or a liquidity shock (a private-value shock).[16] With probability η, the common value of the asset experiences an innovation and becomes $\tilde{v} = v_0(1 + \tilde{\sigma})$, where $\tilde{\sigma} = \pm\sigma$ with the same probability. Without a loss of generality, we normalize the initial value of the asset to $v_0 = 1$.

There is a continuum of risk-neutral *informed traders* with a unit measure. When a common-value shock hits the asset, they immediately observe the realized value of the shock, sequentially arrive at the market, and trade the asset by choosing one of two trading venues (defined below). As in conventional market microstructure theory (e.g., Glosten and Milgrom, 1985), an informed trader sends a single-unit market order after deciding on her trading venue.

With probability $1 - \eta$, a shock hits the private value of *liquidity traders*. Liquidity traders are impatient investors with no material information. They are motivated by factors like hedging needs, margin constraints, and other immediate borrowing and lending requirements. A private-value shock triggers their needs for immediacy and

[16] See, for example, Menkveld and Zoican (2017) and Brolley and Zoican (2020) for models with these shocks as a trigger of transactions.

makes them want to trade. Following Zhu (2014), mass z_{buy} (resp. z_{sell}) of liquidity buy (resp. sell) market orders arrive at the market, where z_{buy} and z_{sell} are random variables independently and identically distributed on $[0, \bar{z})$ with mean $0.5z$. For $i \in \{buy, sell\}$, the random trading size z_i can be thought of as the aggregate orders from n liquidity buyers (each indexed by k), whose trading sizes $\{z^n_{i,k}\}$ are iid random variables and add up to z_i. In the limit of $n \to \infty$, individual trading size $z^n_{i,k}$ becomes infinitesimal and has no impact on the distribution of the aggregate order size z_i. Namely, the joint distribution of z_{buy} and z_{sell} conditional on each trader's order size $z^n_{i,k}$ becomes the same as the unconditional joint distribution of z_{buy} and z_{sell}.[17]

We assume that liquidity traders must decide on their venue at $t = 0$, i.e., before they enter the market. This is because they are unsophisticated retail investors, and maintaining multiple accounts on both exchanges (or subscribing to a smart order router) is costly.[18] Appendix B relaxes this assumption and allows them to choose their trading venues contingent on the sign of a private-value shock.

There is also a continuum of uninformed *market makers* (*liquidity providers*) with a sufficiently large measure. At the beginning of the game, competitively many market makers either post a single-unit limit order on a limit-order market or lock one unit of assets in the liquidity pools of an automated market. At the end of $t = 1$, information about $\tilde{v}$ becomes public, and market makers revoke (reprice) their limit orders and withdraw liquidity from the pools to avoid an adverse selection cost and an impermanent loss.[19]

[17] See the discussion in Zhu (2014) for the formal microfoundation and convergence results at $n \to \infty$.

[18] As of July 2021, only a limited number of cryptocurrency exchanges provide order routine services across CEXs and DEXs. An investor may trade via institutional brokers, but a large portion of cryptocurrency trades are done directly by retail investors.

[19] Subsection 6.2 justifies liquidity removal by market makers by considering exogenous arbitragers.

Figure 3: Timeline of the game

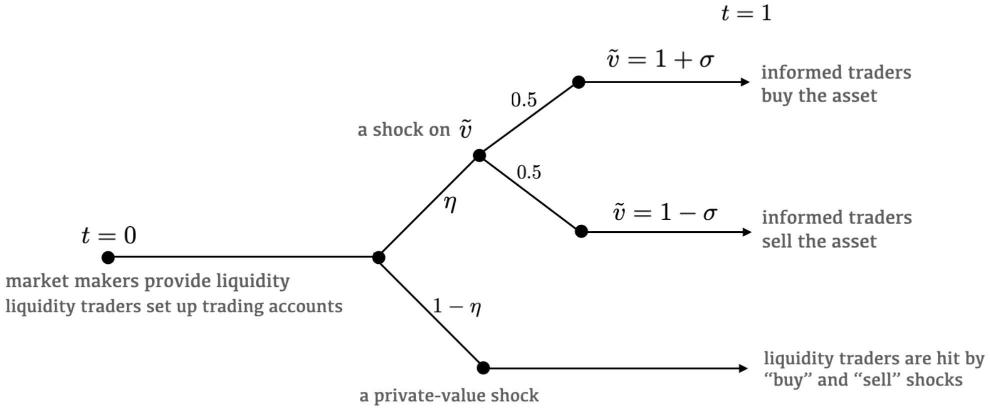


Figure 3 illustrates the timeline of the game and possible outcomes of the trigger event.

Exchange platforms. There are two exchange platforms: a CEX and a DEX. The CEX is a traditional centralized exchange and operates with a continuous limit order book (LOB). It retains custody of trader funds and is based on a centralized matching algorithm using high-speed information processors. Thus, it provides ultra-fast trade execution, causing almost no delays.[20]

In contrast, the DEX handles transactions via a CFMM. As explained in Subsection 2.2, the platform generates *liquidity pools* by using the provided assets, and a liquidity taker trades against the pools. The execution price is determined by the CFMM algorithm instead of quotes by market makers.

Trading with the CFMM involves smart contracts on the Ethereum blockchain and its throughput is lower than the CEX, causing a delay in completing a transaction.[21] Following Zhu (2014), a delay in trade execution weighs negatively on the private

[20] Delays in order execution on centralized exchanges are scaled by microseconds or nanoseconds and are almost negligible in this paper.

[21] Aside from the delays in executing transactions on decentralized exchanges, there is a different source of delays that is common in DEXs and CEXs. If a user has digital tokens and wants to use them

utility of liquidity traders, as they are impatient and eager to fulfill their trading needs immediately.[22] In the model, a liquidity trader on the DEX incurs $\gamma\sigma$ of delay costs per unit of trade, where γ represents heterogeneous aversion toward a delay (or need for immediacy) and $\gamma \sim U[0,1]$. The delay cost is proportional to asset volatility σ. It can be seen as margin constraints or unmodeled risk aversion (e.g., Brunnermeier and Pedersen, 2009; Zhu, 2014).

Although the following discussion regards γ as the delay cost, this interpretation is not essential to our model. Indeed, we can relax the assumption and allow γ to take negative values, e.g., $\gamma \sim U[-\underline{\gamma}, 1]$, with an additional parameter assumption. $\gamma < 0$ means that some traders prefer the DEX to the CEX due to, for example, cyber security risks on the CEX. Thus, the model can incorporate a variety of differences between these exchanges in a reduced form. The modeling assumption is innocuous to traders' behavior in the equilibrium but leads to different welfare implications, as discussed in Subsection 5.4.

Solving the model. We solve the model by taking steps backward. We first analyze traders' behavior given the market liquidity. Afterward, we consider endogenous liquidity supply. Subsection 3.5 endogenizes liquidity on the CEX with that on the DEX being fixed, and Section 5 considers endogenous DEX liquidity.

Traders' behavior is described by their platform choice. Measure β_{buy} (resp. β_{sell}) of informed traders buy (resp. sell) the asset on the DEX when $\tilde{v} = 1 + \sigma$ (resp. $\tilde{v} = 1 - \sigma$). It turns out that informed traders behave asymmetrically depending on the trade direction, $\beta_{buy} \neq \beta_{sell}$, due to the convex nature of the CFMM. We denote the

to make a payment in the real world, she converts them into fiat currency (e.g., USD) by trading on a centralized exchange and transfers the funds to her bank account. Although buying fiat currency on the CEX takes almost no delays, transferring funds can cause some delays. However, this does not affect the model description because a user incurs this delay no matter where she obtains her tokens.

[22] Imposing a delay cost on informed traders does not change the qualitative results but adds complexity to our analyses.

fraction of liquidity traders on the DEX as $\alpha \in [0, 1]$, which is not contingent on the sign of a private-value shock, as they decide on the trading venue at $t = 0$ (see Appendix B for the case with asymmetric α).

For simplicity, our model does not incorporate fees, such as maker/taker fees and Ethereum gas fees on the DEX. Also, the possibility of dynamic arbitrage trading after informed/noise trading is beyond the scope of our one-shot trading environment. These points are further discussed in Subsection 6.2.

3.2 Trading on the CEX

The partial equilibrium on the CEX with the limit order book is standard and follows the model by Zhu (2014). We denote the equilibrium bid and ask prices as

$$\text{Ask} = 1 + a, \ \text{Bid} = 1 - b.$$

For brevity, a and $-b$ are sometimes used to refer to the ask and bid prices. Also, the (effective) bid-ask spread is defined as $S = a + b$. Following market microstructure theory, we guess that the bid and ask prices depend on the signal-to-noise ratio of order flows and denote them as $a = a(\beta_{buy}, \alpha)$ and $b = b(\beta_{sell}, \alpha)$.

Accordingly, the expected profits for an informed trader who trades on the CEX are given by

$$\pi_I^C(\tilde{\sigma}) = \begin{cases} \sigma - a(\beta_{buy}, \alpha) & \text{if } \tilde{\sigma} = +\sigma \text{ and buys the asset,} \\ \sigma - b(\beta_{sell}, \alpha) & \text{if } \tilde{\sigma} = -\sigma \text{ and sells the asset.} \end{cases} \tag{1}$$

Note that the informed trader's profits are conditional on the realized value of $\tilde{\sigma}$. Similarly, a liquidity trader's *ex-post* profits per unit of trading on the CEX are given by[23]

$$\pi^C_{L,k} = \begin{cases} -a(\beta_{buy}, \alpha) & \text{if } k = buy, \\ -b(\beta_{sell}, \alpha) & \text{if } k = sell, \end{cases} \tag{2}$$

where subscript k indicates whether the private-value shock induces a trader to buy or sell the asset. Since each liquidity trader expects to buy and sell with the same probability, her *ex-ante* expected profit per unit of trading is

$$\frac{1}{2}\mathbb{E}\left[\sum_{k=buy,sell} \pi^C_{L,k}\right] = -\frac{S}{2}. \tag{3}$$

Namely, a liquidity trader expects to pay the (half) bid-ask spread on the CEX.

3.3 Trading on the DEX

Constant function market makers. In this section, we assume that the liquidity providers supply an exogenous and sufficiently large amount of the asset, denoted as X (see Section 5 for endogenous X). The initial size of the cash pool is denoted as C, and the following discussion derives C as a function of the initial asset pool, X, via the non-arbitrage condition at the beginning of the game.

If an incoming market order trades δ units of the asset ($\delta > 0$ means a buy order)

[23] We implicitly assume that a liquidity trader obtains private utility u if she fulfills her trading needs, with u being sufficiently large (e.g., $u > 1 + \sigma$). Therefore, all liquidity traders participate in the market upon being hit by a shock. u does not affect the equilibrium conditions because a liquidity trader obtains it no matter which platform she uses.

at cumulative execution price P, the state of the liquidity pools changes as follows.

$$C \to C' = C + P\delta \tag{4}$$

$$X \to X' = X - \delta. \tag{5}$$

The CFMM is defined by function $f : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$, which sets the execution price by requiring the post-trade state of the pools to satisfy[24]

$$f(C, X) = f(C + P\delta, X - \delta). \tag{6}$$

We impose the following regularity conditions on f to pin down a unique price.[25]

Condition 1. *The CFMM function $f : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ with initial liquidity pool (C, X) satisfies the following:*
(i) f is differentiable, $\frac{\partial^2 f}{\partial c \partial x}$ exists, and $f_c(c, x)$ and $f_x(c, x)$ are positive for all $c, x > 0$;
(ii) If (c, x) is on function f, there exists $h : \mathbb{R}_{++} \to \mathbb{R}_{++}$ such that $c = h(x; C, X)$ and h is decreasing in x; and
(iii) $\{(c, x) | f(c, x) > k\}$ is a strictly convex set for all $k > 0$.

Note that the conditions above are satisfied by most of CFMMs in the real world. For example, a CPMM is the special case with $f(C, X) = CX$ and satisfies all the regularity conditions.

To establish the model with the coexisting exchanges, we first assume that no arbitrage exists at the beginning of the game.

Assumption 1. *At the beginning of the trading game, there is no arbitrage, and an infinitesimal trade cannot make a strictly positive profit.*

[24] The assumptions that f is convex and continuously differentiable are sufficient for the following result. For the proof, we derive weaker conditions for f in Appendix C.

[25] Appendix C provides additional technical conditions to rule out multiple equilibria and unrealistic behavior of the equilibrium price.

Note that an infinitesimal trade ($\delta \to 0$) is executed at the marginal price[26]

$$p_0 \equiv \frac{f_x(C, X)}{f_c(C, X)}, \tag{7}$$

where we denote the partial derivative of $f(c, x)$ with respect to $j = c, x$ as f_j. Therefore, the non-arbitrage condition is given by $p_0 = \mathbb{E}[\tilde{v}] = 1$.

For a general CFMM, the non-arbitrage condition implies that there exists a differentiable and increasing function, denoted as $g : \mathbb{R}_{++} \to \mathbb{R}_{++}$, such that $C = g(X)$. This function specifies the initial condition of the cash pool and the CFMM constant k given the size of the asset X that liquidity providers intend to supply.[27]

With a CPMM, for example, the price in (7) is given by $p_0 = C/X$, and the non-arbitrage condition requires the initial state to satisfy $C = g(X) = X$. Also, this condition pins down the CPMM constant as $k = CX = X^2$. For simplicity, we denote the following results by using X rather than denoting $(C, X) = (g(X), X)$.

Secondly, we characterize a set of "reachable states" for liquidity pools, (c, x), on the CFMM curve. Namely, if state (c, x) is on the CFMM curve with the initial condition $(C, X) = (g(X), X)$, the value of c is expressed by using monotonically decreasing function $h : \mathbb{R}_{++} \to \mathbb{R}_{++}$ as

$$c = h(x; X). \tag{8}$$

Equation (8) draws a convex curve of the CFMM. In the case with a CPMM, the above equation is $c = h(x; X) = \frac{X^2}{x}$ because the CPMM curve is specified by $k = XC = X^2$ due to the non-arbitrage condition. In what follows, we denote $c = h(x)$ unless otherwise specified.

By using the above functions, we can derive the marginal and expected cumulative prices for a trade. Proofs for the following results are provided in Appendix C.

[26] Taking the first-order derivative of equation (6) with respect to δ and setting $\delta \to 0$ yield p_0.

[27] As explained in Subsection 2.2, $C = g(X)$ determines the rule of liquidity provision when market makers endogenously supply (C, X) in Section 5.

Lemma 1. *Given the initial state of the liquidity pools, $(g(X), X)$, and function $c = h(x)$ in (8), the expected price for a trade with size $\delta \neq 0$ is given by*

$$P(\delta, X) = \frac{1}{\delta} \int_0^{\delta} \frac{f_x(h(X - \tilde{\delta}), X - \tilde{\delta})}{f_c(h(X - \tilde{\delta}), X - \tilde{\delta})} d\tilde{\delta}. \tag{9}$$

Also, the marginal price for an infinitesimal trade after δ units of trade is made is given by

$$p(\delta, X) = \frac{f_x(h(X - \delta), X - \delta)}{f_c(h(X - \delta), X - \delta)}. \tag{10}$$

The marginal price function, p, satisfies the following conditions.[28]
(i) p is increasing in δ;
(ii) p is decreasing in X if, and only if, $\delta > 0$;
(iii) p is differentiable with respect to both elements, and $\frac{\partial^2 p}{\partial \delta \partial X} < 0$; and
(iv) $p_\delta(m, X) > p_\delta(-m, X)$ holds for all $m, X > 0$

For a fixed initial liquidity X, Condition 1 essentially suggests the monotonicity and convexity of the marginal price function with respect to trade size δ.

Firstly, the more the trader intends to buy (resp. sell), the higher (resp. lower) the marginal execution price becomes (condition [i]). Moreover, conditions (ii) and (iii) imply that ample liquidity lowers the marginal price given the size of a trade when a trader is buying the asset ($\delta > 0$), while it increases the marginal price for a seller ($\delta < 0$). In other words, a larger liquidity pool (X) makes the market deeper, and a market order of a given size has a smaller price impact. Hence, it is appropriate to use the pool size (X) as the measure of liquidity on the DEX. Finally, condition (iv) implies the convexity of the price function regarding δ, though the condition is weaker than the convexity.

[28] *In addition, we can show that (v) $A(\delta, X) \equiv |p(\delta, X) - 1|$ is log-submodular in (δ, X) for $\delta \neq 0$. We use this condition in the formal proof in Appendix C.*

With a CPMM, for example, the expected and marginal prices are given by

$$P(\delta, X) = \frac{X}{X-\delta} \text{ and } p(\delta, X) = \frac{X^2}{(X-\delta)^2}. \tag{11}$$

It is straightforward to check that Lemma 1 holds because $\frac{\partial p}{\partial \delta} = \frac{2X^2}{(X-\delta)^2} > 0$, $\frac{\partial p}{\partial X} = \frac{-2\delta X}{(X-\delta)^3}$, and $\frac{\partial^2 p}{\partial X \partial \delta} = \frac{-2X(X+2\delta)}{(X-\delta)^4} < 0$.

Profits of informed traders on the DEX. Informed traders arrive sequentially to the market, and each observes the current state of the liquidity pools to learn how much informed trading has already been conducted. If $\tilde{\beta}$ measure of informed traders have already traded on the DEX, the next arriving informed trader executes her order at marginal price $p(\tilde{\beta}, X)$. As a result, the informed trader's marginal profit on the DEX, given that $\tilde{\beta} \in \{\tilde{\beta}_{buy}, -\tilde{\beta}_{sell}\}$ measure of other informed traders have completed their transactions, is

$$\pi_I^D(\tilde{\sigma}, \tilde{\beta}) = \begin{cases} 1 + \sigma - p(\tilde{\beta}_{buy}, X) & \text{if } \tilde{\sigma} = +\sigma \\ p(-\tilde{\beta}_{sell}, X) - (1 - \sigma) & \text{if } \tilde{\sigma} = -\sigma. \end{cases} \tag{12}$$

Note that each informed trader trades one unit of the asset and has an infinitesimal measure. Thus, she is concerned about the marginal cost (or return) of trading, p, relative to the return (or cost), $\tilde{v} = 1 + \tilde{\sigma}$.

Profits for liquidity traders on the DEX. Conditional on the realization of the random trading size, a liquidity trader with γ expects to obtain the following interim profits per unit.

$$\mathbb{E}_{\Delta z}[\pi_{L,k}^D(\gamma)] = \begin{cases} 1 - \mathbb{E}_{\Delta z}[P(\alpha \Delta z, X)] - \gamma\sigma & \text{if } k = buy, \\ \mathbb{E}_{\Delta z}[P(\alpha \Delta z, X)] - 1 - \gamma\sigma & \text{if } k = sell. \end{cases} \tag{13}$$

Note that the mean price, P, matters from the interim perspective, as the trader does not know the timing of her order execution when she decides on her venue. The aggregate order size, $\Delta z = z_{buy} - z_{sell}$, is uncertain for each liquidity trader, and $\mathbb{E}_{\Delta z}$ is the expectation over Δz. Since each liquidity trader is infinitesimal, observing her own trading size does not affect her inference regarding the aggregate order size, z_i.

Importantly, the expected price impact of liquidity trading on the DEX tends to be weaker than that of informed trading. This is because liquidity traders conduct random trading, and buy and sell orders tend to cancel out. In contrast, informed traders trade based on the same information and cluster on the same side of the market.

3.4 Platform Choice

Given liquidity on the DEX and the CEX, the proportions of informed and liquidity traders who participate in the DEX, $\{(\beta_i)_{i=buy,sell}, \alpha\}$, are determined so that (i) the informed traders become indifferent between the two platforms, and (ii) the liquidity traders are differentiated by comparing the *ex-ante* expected profits on each exchange. We search for an equilibrium where the DEX and the CEX coexist and have a strictly positive share, i.e., $\beta_i, \alpha \in (0, 1)$.

Firstly, informed traders choose their platforms so that the marginal costs on both exchanges become identical.

$$1 + a(\beta_{buy}, \alpha) = p(\beta_{buy}, X), \tag{14}$$

$$1 - b(\beta_{sell}, \alpha) = p(-\beta_{sell}, X). \tag{15}$$

In both equations, the LHS represents the marginal trading cost on the CEX, while the RHS is that on the DEX. If the above equations do not balance, marginal informed traders switch their venue, and β_i adjusts so the equations hold. Indeed, the next section shows that such an equilibrium exists and is unique and stable.[29]

Secondly, a liquidity trader with cost parameter γ participates in the DEX if, and only if, the expected profit (conditional on γ) from trading on the DEX surpasses that from trading on the CEX:[30]

$$\mathbb{E}\left[\sum_{k=buy,sell} \pi^{D}_{L,k}(\gamma)\right] \geq \mathbb{E}\left[\sum_{k=buy,sell} \pi^{C}_{L,k}\right] = -S, \tag{16}$$

where the last equality uses (3). By using (13), the above inequality is reduced to

$$\gamma \leq \gamma^* \equiv \frac{S(\beta_{buy}, \beta_{sell}, \alpha)}{2\sigma}.$$

The LHS is the expected trading cost on the DEX. Since a liquidity trader buys and sells with the same probability, the execution price does not affect the expected cost. However, the delay cost matters because a liquidity trader on the DEX bears it regardless of the trading direction. In contrast, the RHS represents the expected trading cost on the CEX, i.e., the bid-ask spread. Since $\gamma \sim U[0,1]$, we obtain

$$\alpha = \Pr(\gamma < \gamma^*) = \frac{S(\beta_{buy}, \beta_{sell}, \alpha)}{2\sigma}. \tag{17}$$

Liquidity traders with a relatively small γ prefer trading on the DEX, as this exchange provides a lesser expected price impact with a lower delay cost. The opposite is true if γ is relatively large.

[29] Since liquidity takers choose their trading venues by comparing the *ex-ante* expected prices, it is possible that the *ex-post* price on the DEX deviates from the asset's value. This deviation may cause further arbitrage trading, and Subsection 6.2 discusses how to incorporate this possibility without changing the results in our one-shot trading environment.

[30] We assume that a liquidity trader participates in the DEX when she is indifferent.

3.5 Liquidity on the Centralized Limit-Order Market

We investigate whether conditions (14) to (17) have a unique set of solutions by specifying the bid and ask prices on the CEX. On the LOB, one of the competitive market makers is active on the equilibrium path due to the price-time priority, i.e., one market maker can post her limit order on the top of the book and be matched with an incoming trade. We call this trader the *CEX market maker*. Since other potential market makers are willing to post a better price off the equilibrium path, the CEX market maker posts a competitive quote (see Zhu, 2014 and Baldauf and Mollner, 2020 for the same setting).

Given the venue choice of traders, the expected profits for the CEX market maker from posting a and b on each side of the market are

$$\pi^C_{M,ask} = \frac{1}{2}\left[\eta(1-\beta_{buy})(a-\sigma) + (1-\eta)(1-\alpha)za\right],$$
$$\pi^C_{M,bid} = \frac{1}{2}\left[\eta(1-\beta_{sell})(b-\sigma) + (1-\eta)(1-\alpha)zb\right].$$

In both equations, the first term represents a trade with an informed trader, which occurs with expected mass $\frac{1}{2}\eta(1-\beta_i)$, and the second term shows the case of liquidity trading, which occurs with expected mass $\frac{1}{2}(1-\eta)(1-\alpha)z$. Following Zhu (2014), we focus on the equilibrium in which the CEX market maker breaks even on both sides of the market, leading to the following competitive bid and ask prices.

$$a = a(\beta_{buy}, \alpha) = \sigma\frac{(1-\beta_{buy})\eta}{(1-\beta_{buy})\eta + (1-\eta)(1-\alpha)z}, \tag{18}$$
$$b = b(\beta_{sell}, \alpha) = \sigma\frac{(1-\beta_{sell})\eta}{(1-\beta_{sell})\eta + (1-\eta)(1-\alpha)z}. \tag{19}$$

The above prices are potentially asymmetric, as we allow $\beta_{buy} \neq \beta_{sell}$. The spread positively (resp. negatively) reacts to the intensity of informed (resp. liquidity) trading, because it exacerbates (resp. mitigates) the adverse selection cost for a market maker.

4 Partial Equilibrium with Exogenous DEX Liquidity

We first define and analyze the (partial) equilibrium with exogenous DEX liquidity, X, as it helps us endogenize X in Section 5.

Definition 1. *The partial equilibrium with exogenous DEX liquidity (X) is defined by the proportions of informed and liquidity traders on the DEX, $\{(\beta_i)_{i=buy,sell}, \alpha\}$, the bid-ask prices, (a, b), and the marginal execution prices on the DEX, p, such that (i) the marginal informed traders satisfy indifference conditions (14) and (15), (ii) the liquidity traders are differentiated by equation (17), (iii) the bid and ask prices are given by* (18) and (19), *and (iv) the price on the DEX is provided by (10) given the initial liquidity pool $(g(X), X)$.*

With the bid and ask prices given by (18) and (19), the indifference conditions (14) to (17) pin down the equilibrium mass of traders on the DEX given X. To obtain an interior solution, we assume that the expected mass of liquidity trading is sufficiently large.

Assumption 2. *The expected size of liquidity trading satisfies $z > z^* \equiv \frac{\eta}{1-\eta}$.*

Assumption 2 guarantees that market makers do not face extremely severe adverse selection and the market does not break down.

Liquidity traders. Based on (17), (18), and (19), liquidity traders' platform choice is characterized by

$$2\alpha = \frac{(1-\beta_{buy})\eta}{(1-\beta_{buy})\eta + (1-\eta)(1-\alpha)z} + \frac{(1-\beta_{sell})\eta}{(1-\beta_{sell})\eta + (1-\eta)(1-\alpha)z}, \tag{20}$$

where the RHS is the normalized bid-ask spread.

Lemma 2. *(i) Equation (20) obtains a unique interior solution of the fraction of the DEX liquidity traders, α, as a function of $(\beta_{buy}, \beta_{sell}, X)$. We denote it as $\alpha^* = \alpha^*(\beta_{buy}, \beta_{sell}, X)$. (ii) With X being fixed, α^* is monotonically decreasing in β_{buy} and β_{sell}.*

The fraction of liquidity traders on the DEX decreases as the measure of informed traders on this exchange increases. When informed traders migrate away from the CEX to the DEX (β_i increases), the bid-ask spread on the CEX tightens. Since the execution price and delay costs for liquidity trading on the DEX are not directly affected by β_i, the CEX becomes more attractive for the liquidity traders, leading to a lower α^*.

Informed traders. Analogously, the indifference conditions for informed buyers and sellers are given by

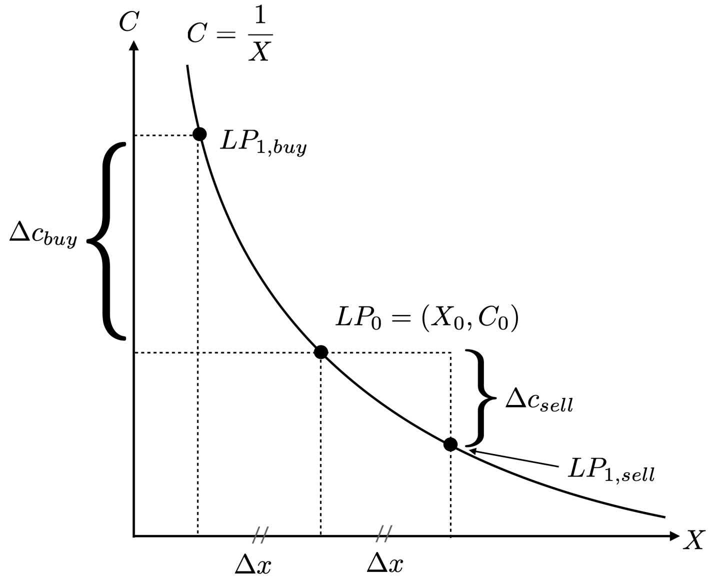
$$p(\beta_{buy}, X) = 1 + \sigma \frac{(1-\beta_{buy})\eta}{(1-\beta_{buy})\eta + (1-\eta)(1-\alpha)z}, \tag{21}$$

$$p(-\beta_{sell}, X) = 1 - \sigma \frac{(1-\beta_{sell})\eta}{(1-\beta_{sell})\eta + (1-\eta)(1-\alpha)z}. \tag{22}$$

Lemma 3. *(i) Equations (21) and (22) yield unique measures of informed buyers and sellers on the DEX as functions of (α, X). We denote them as $\beta_i^* = \beta_i^*(\alpha, X)$ for $i \in \{buy, sell\}$.*
(ii) With X being fixed, β_i^* is monotonically increasing in α.
(iii) $\beta_{buy}^* < \beta_{sell}^*$ for all $\alpha \in (0,1)$ and X.

For $i \in \{buy, sell\}$, β_i^* is a unique and stable solution for each indifference condition in $\beta \in (0,1)$. The trading intensity of the informed traders exhibits anticipated reactions to a change in liquidity traders' behavior. If a larger set of liquidity traders participate in the DEX (α increases), it exacerbates adverse selection for market makers on the CEX. It widens the bid-ask spread, encouraging more informed traders to participate in the DEX (β_i^* increases).

Figure 4: Asymmetric price impact



Note: This figure describes the price impact of buy and sell orders of the same size, Δx, along the CPMM curve.

One of the novel features of the CFMM algorithm emanates from asymmetry in execution prices due to its convexity (see Condition 1). Figure 4 illustrates the intuition behind point (iii) by taking the CPMM as an example. A buy order with size $\delta = \Delta x > 0$ moves the initial liquidity pool LP_0 upward to $LP_{1,buy}$, while a sell order of the same size moves LP_0 downward to $LP_{1,sell}$, both along the curve $C = X^{-1}$. Since the curve is convex, the buy order requires a larger adjustment along the y-axis than the sell order, $\Delta c_{buy} > \Delta c_{sell}$.[31] The convexity of the CFMM implies that the execution price is determined so that adding liquidity to the pools bears a smaller price impact than consuming it. Hence, an innovation in the asset's value induces a dispropor-

[31] Although the curve $y = k/x$ is symmetric around $x = y$, the reaction of y is asymmetric to $x \pm d$ as long as $d > 0$. Moreover, the asymmetric price reaction does not mean the existence of an arbitrage opportunity. Buying δ and selling the same amount simply push the liquidity pools back to their original position, leading to no profits. This logic is true even if we switch the role of cash and the asset.

tional reaction of informed sellers to informed buyers, leading to $\beta^*_{buy} < \beta^*_{sell}$. This asymmetry generates the following result.

Corollary 1. *The midpoint quote on the CEX is biased and is higher than the expected value of the asset, i.e.,* $\frac{Ask+Bid}{2} > \mathbb{E}[\tilde{v}]$.

Further implications regarding the informativeness of an order flow and the bid-ask spread are discussed later.

By using the results in Lemmas 2 and 3, we can express the partial equilibrium measures of informed and liquidity traders on the DEX as functions of the DEX liquidity, X.

$$\alpha^*(X) \equiv \alpha^*(\beta^*_{buy}(\alpha^*, X), \beta^*_{sell}(\alpha^*, X), X), \tag{23}$$

$$\beta^*_i(X) \equiv \beta^*_i(\alpha^*(\beta^*_{buy}, \beta^*_{sell}, X), X). \tag{24}$$

Proposition 1. *In the equilibrium, there is a unique set of* $(\alpha^*(X), \beta^*_{buy}(X), \beta^*_{sell}(X))$ *that solves equations (20)-(22), and the solutions are stable.*

4.1 Liquidity Impact of Automated Market Makers

Analyzing the above equations answers an important question: does additional liquidity on the DEX improve or harm liquidity on the CEX? Since the bid-ask spread on the CEX is determined by the signal-to-noise ratio of a trade, we must investigate the reaction of $(\beta^*_{buy}, \beta^*_{sell})$ relative to that of α^* to a change in DEX liquidity, X.

Proposition 2. *(i) The measure of informed buyers on the DEX,* β^*_{buy}*, is increasing in* X.
(ii) The expected measure of informed traders on the DEX, $\frac{\beta^*_{buy}+\beta^*_{sell}}{2}$*, is increasing in* X.
(iii) The proportion of the liquidity traders on the DEX, α^**, is decreasing in* X.
(iv) The bid-ask spread on the CEX is decreasing in X.

Proposition 2 shows that informed traders are inclined to participate in the DEX when it becomes more liquid, whereas liquidity traders tend to use the CEX.

Firstly, informed traders are concerned about the price impact on the DEX, which is decreasing in X. Thus, larger liquidity pools marginally reduce the cost of informed trading on the DEX and attract more informed traders. In turn, more active informed traders on the DEX mitigate the adverse selection cost for the CEX market maker, and the bid-ask spread declines.

Secondly, a change in X does not directly affect liquidity traders' behavior, as the execution price on the DEX does not matter to them in expectation. Hence, facing a narrower bid-ask spread on the CEX triggered by the migration of informed traders, more liquidity traders move to the CEX.

This process involves a decline in the bid-ask spread or improved market liquidity on the CEX, as demonstrated by point (iv) in Proposition 2. Therefore, our model suggests that liquidity on the DEX complements that on the CEX. This result not only helps us derive the general equilibrium with endogenous X in the next section but also provides testable implications discussed in Section 6.

5 Equilibrium with Endogenous DEX Liquidity

This section considers liquidity provision by market makers on the DEX. Prior to the trading game (at $t = 0$), each market maker decides whether to supply one unit of the asset to the liquidity pool. Since the market-making sector on the CEX is competitive and yields no profit in expectation, it works as an outside option for market makers on the DEX.

To guarantee the existence of the equilibrium, we assume the presence of *passive* liquidity providers on the DEX. They provide some exogenous amount of liquidity, $x_{passive} > \underline{x} \equiv \frac{\bar{z}\eta}{z(1-\eta)}$, and stay inactive. This assumption is to avoid market break-down: even if the active liquidity providers supply zero liquidity, the DEX has a positive amount of liquidity that can absorb the potential size of liquidity-taking orders.[32]

The aggregate supply of the asset is defined as $X \equiv x_{passive} + 1 \times m$, where m denotes the measure of active market makers. We search for the equilibrium value of X (or, equivalently, m) in the following sections by focusing on the pairs of endogenous variables $(X, \alpha, \beta_{buy}, \beta_{sell})$ that satisfy the equilibrium conditions.[33]

5.1 Market Makers' Profits on the DEX

When a market maker locks the asset, she must supply c units of cash into the pool, obtaining $w = \frac{c+1}{C+X}$ share of the aggregate pools.[34] She must follow this rule so the non-arbitrage condition remains true. With a trade of size δ, the post-trade liquidity pools have C' and X' in equations (4) and (5). After a trade, the market maker earns w share of (C', X'), realizing the difference from the initial cost as her net profit.

Impermanent loss. With probability η, information-driven traders take liquidity where they are buying and selling with measures β_{buy} and β_{sell}, respectively, with the same probability. Conditional on these events, the expected profit for a market maker, net

[32]Otherwise, even an infinitesimal trade can have a large price, leading to a corner solution for either β_i or α. The existence of persistent liquidity providers is observed in the real market. See, for example, Lehar and Parlour (2021).

[33]In particular, the following analyses focus on the variables, $(X, \alpha, \beta_{buy}, \beta_{sell})$, that satisfy the equilibrium conditions on the marginal price and informed trading: $p(\beta_{buy}(X), X) < 1 + \sigma$ and $p(-\beta_{sell}(X), X) > 1 - \sigma$.

[34]The amount of cash supply is different among market makers and depends on the assumption regarding the timing of liquidity supply. If we assume that market makers sequentially supply liquidity, each of them faces different states of liquidity pools. For example, if market maker i supplies x units of the asset when other market makers have already injected X_0 and C_0, the current state of liquidity pools is $(x_{passive} + X_0, c_{passive} + C_0)$. Then, her cash supply must satisfy $c_i = g(x_{passive} + X_0 + x)$, which depends on (X_0, C_0). It turns out that c_i does not affect the equilibrium conditions (Proposition 5), and thus we do not specify c_i for each liquidity provider.

of the initial cost of injecting assets, is given by π^D_{IT} below.

$$\pi^D_{IT} = \frac{w}{2}\Bigg[\overbrace{g(X) + P(\beta_{buy}, X)\beta_{buy} + (1+\sigma)(X - \beta_{buy})}^{\text{if } \tilde{\sigma} = +\sigma}$$

$$+\overbrace{g(X) - P(-\beta_{sell}, X)\beta_{sell} + (1-\sigma)(X + \beta_{sell})}^{\text{if } \tilde{\sigma} = -\sigma}\Bigg] - (c+1)$$

$$= \frac{w}{2}\left[\left(P(\beta_{buy}, X) - (1+\sigma)\right)\beta_{buy} + \left((1-\sigma) - P(-\beta_{sell}, X)\right)\beta_{sell}\right].$$

The first and second lines represent the post-trade net value of the liquidity pools, which involves either a positive or negative shock on $\tilde{v}$ and informed trading. Since market makers absorb cumulative trading volumes, their profit involves the expected price, P, rather than the marginal price, p.

Proposition 3. *When a trade is triggered by a common-value shock, (i) the market maker's expected net profit on the DEX is negative, i.e.,* $\pi^D_{IT} < 0$*. (ii) With X being fixed,* π^D_{IT} *is decreasing in* β_i*.*

The negative profit from informed trading is called *impermanent loss* (see, for example, Angeris and Chitra, 2020). The fact that liquidity is taken by an informed trader implies that the value of the liquidity pools inevitably declines. This is because an informed trader always subtracts a more valuable asset from the liquidity pools by adding a less valuable asset. This result highlights the similarity of the CFMM to market making on the limit order book, where informed trading involves adverse selection for market makers due to information asymmetry (e.g., Glosten and Milgrom, 1985; Kyle, 1985)

Profits from noise. Liquidity traders cause noise trading, i.e., their behavior is independent of the value of the asset. This results in market orders with stochastic size $\Delta z = z_{buy} - z_{sell}$. The expected net profit of a market maker conditional on a private-value shock is given by

$$\pi^D_{LT} = w\mathbb{E}_{\Delta z}\left[g(X) + P(\alpha\Delta z, X)\alpha\Delta z + (X - \alpha\Delta z)\right] - (c+1)$$
$$= w\mathbb{E}_{\Delta z}\left[P(\alpha\Delta z, X)\alpha\Delta z\right].$$

Proposition 4. *(i) When a trade is triggered by a private-value shock, the market maker's expected net profit on the DEX is positive, i.e.,* $\pi^D_{LT} > 0$.
(ii) π^D_{LT} *is increasing in* α *and decreasing in* X.

As in limit-order markets, market makers on the DEX gain from trading with liquidity traders because uninformed liquidity trading improves the value of the liquidity pools. The strictly positive profits emanate from the liquidity pool of cash.[35] Since the execution price adjusts the post-trade liquidity pools along the convex curve $f(C, X)$, Jensen's inequality implies that $\mathbb{E}[P(\alpha\Delta z)\alpha\Delta z] > 0$. Therefore, the positive impact of liquidity trading is hard-wired in the CFMM's convex pricing algorithm and works as an implicit reward for liquidity providers.[36] This profit mechanism is magnified when the volatility of liquidity trading (α) is large. In contrast, greater liquidity (X) diminishes the variation in liquidity trading and reduces π^D_{LT}.

The profit mechanism in Proposition 4 is absent in the literature on automated market makers. The existing theory has analyzed how the price in an automated market converges to an exogenous reference price, where arbitragers facilitate this convergence. We introduce liquidity or noise traders following the microstructure literature (e.g., Grossman and Stiglitz, 1980; Black, 1986; DeLong et al., 1990) and show that they play an important role in motivating liquidity provision even without fee rebates to market makers.

[35]Since liquidity buy and sell orders are netted out and are independent of $\tilde{\sigma}$, liquidity trading does not change the expected value of the asset pool ($\mathbb{E}[X'] = X - \alpha\mathbb{E}[\Delta z] = X$).

[36]See Subsection 6.2 for the possibility of subsequent arbitrage trading after the revelation of $\tilde{v}$, which tries to bring the liquidity pools back to their initial state.

5.2 Liquidity Provision on the DEX

The expected profit from providing liquidity on the DEX is the combination of impermanent loss and the profit from noise trading, which occur with probability η and $1-\eta$, respectively.

$$\pi_M^D(X) = \eta \pi_{IT}^D(X) + (1-\eta)\pi_{LT}^D(X). \tag{25}$$

Note that each market maker is infinitesimal and does not incorporate the impact of her liquidity provision on the aggregate liquidity, X, as well as traders' behavior, $\alpha^*(X)$ and $\beta_i^*(X)$ for $i \in \{buy, sell\}$. Since we assume a free entry condition for the market-making sector, the equilibrium size of the liquidity pool is determined by the break-even condition, $\pi_M^D = 0$.

Proposition 5. *Suppose that $(X, \alpha, \beta_{buy}, \beta_{sell})$ satisfy the equilibrium conditions.*
(i) Given β_i and α, the market maker's expected profit is decreasing in X when $\pi_M^D \geq 0$.
(ii) Given β_i and α, there is at most one $X \in [x_{passive}, \infty)$ such that $\pi_M^D(X) = 0$. We denote it as $X^* = G(\alpha, \beta_{buy}, \beta_{sell})$ if such X exists in $X \in [x_{passive}, \infty)$. If a solution for $\pi_M^D(X) = 0$ does not exist in $[x_{passive}, \infty)$, $\pi_M^D(X) < 0$ for all $X \in [x_{passive}, \infty)$. In this case, we define $X^* = x_{passive}(= G(\alpha, \beta_{buy}, \beta_{sell}))$. In both cases, X^* is stable.
(iii) X^* is weakly increasing in α and weakly decreasing in β_i.

Given the behavior of traders, the expected profit for each market maker monotonically decreases with the amount of the asset that is locked (for $\pi_M^D \geq 0$). As a larger number of liquidity providers participate, the individual profit becomes more diluted.

Proposition 5 suggests that there is a unique and stable X^* that solves the break-even condition. As long as $\pi_M^D > 0$, more liquidity providers participate in the DEX and pushes up the value of X, reducing π_M^D. This process continues until $\pi_M^D \leq 0$. In contrast, if $\pi_M^D < 0$, liquidity providers stop supplying liquidity, and X declines until $\pi_M^D \geq 0$ holds. Therefore, the equilibrium is determined by the break-even condition,

$\pi_M^D = 0$, and it is stable.

Moreover, Proposition 5 implies that the amount of liquidity rises with an exogenous change in α, whereas it declines when β_i increases. The intuition follows the traditional discussions on adverse selection: informed trading relative to liquidity trading makes it more costly for market makers on the DEX to supply liquidity.

By using the results in Section 4 and Proposition 5, we define the equilibrium with endogenous DEX liquidity as follows.

Definition 2. *The equilibrium with endogenous DEX liquidity (X) is defined by the proportions of informed and liquidity traders on the DEX, $\{(\beta_i)_{i=buy,sell}, \alpha\}$, DEX liquidity supply, X, the bid-ask prices, (a, b), and the marginal execution prices on the DEX, p, such that (i) the marginal informed traders satisfy indifference conditions (14) and (15), (ii) the liquidity traders are differentiated by equation (17), (iii) the bid and ask prices are given by* (18) and (19)*, (iv) the price on the DEX is given by (10), and (v) the market makers break even on the DEX.*

Mathematically, the equilibrium can be obtained by solving the fixed point problem regarding X^*:

$$X^* = G(\alpha^*(X^*), \beta^*_{buy}(X^*), \beta^*_{sell}(X^*)), \tag{26}$$

where α^* and β^*_i are given by equations (23) and (24), and G is defined by Proposition 5.

Proposition 6. *A unique stable equilibrium with endogenous liquidity supply $(X^*, \alpha^*, \beta^*_{buy}, \beta^*_{sell})$ exists as a solution of the fixed-point problem in (26).*

The existence of equilibrium is guaranteed by the continuity of G, $\alpha^*(X)$, and $\beta^*_i(X)$ and their behavior at $X \to \infty$ (see Appendix C for the formal proof).

Intuition follows from the disproportional reactions of informed and liquidity traders to changes in X attested by Proposition 2. When the DEX obtains more liquidity (X on the LHS of [26] increases), we know from Proposition 2 that the average measure of informed traders on the DEX increases, whereas the measure of liquidity traders declines. These reactions diminish the profit of the DEX liquidity providers. Based on Proposition 5, they hold back from supplying the asset, causing a decline in the RHS of (26). Stability is guaranteed by Proposition 5.

5.3 Comparative Statics

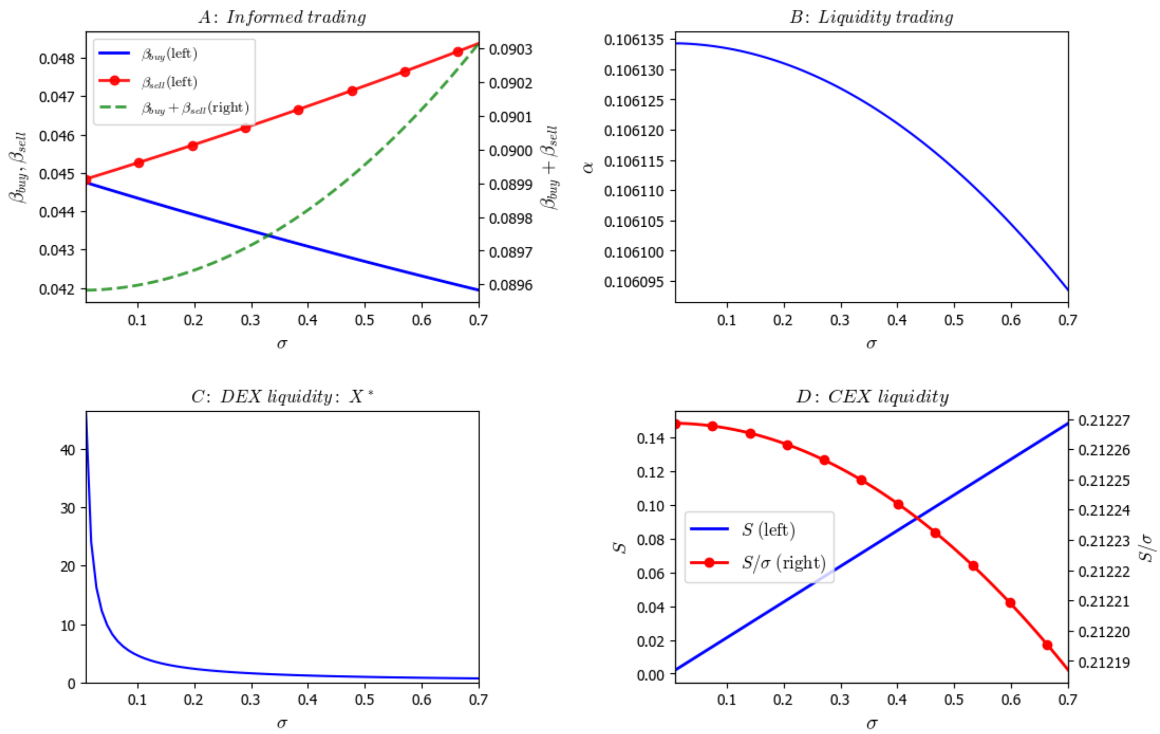
To gauge the joint reaction of the traders' behavior and market liquidity to variations in an exogenous parameter, we numerically analyze the fixed-point problem in (26), as it is not analytically tractable. In what follows, we take the volatility of the asset, σ, as a source of exogenous variation. Qualitative results do not change when we use different values for other deep parameters of the model, η and z.

The following analyses adopt the CPMM algorithm, $f(C, X) = CX$, as the leading example of a CFMM. This is a natural starting point, given that more than 80% of transactions are handled by CPMMs on Uniswap, Sushiswap, and PancakeSwap in the real financial market.

A higher volatility of the asset implies that informed traders possess a greater informational advantage over market makers, and adverse selection worsens both on the DEX and the CEX. This confounds liquidity provision by DEX market makers, leading to a decline in X^*, as well as a wider effective bid-ask spread on the CEX. The above changes directly affect informed trading ($\beta^*_{buy}, \beta^*_{sell}$; via Lemma 3), whereas the measure of liquidity trading (α^*) is only indirectly affected (see equation [20]).

As it becomes more costly to trade on both exchanges, the reaction of β^*_{buy} and β^*_{sell} can be ambiguous. Panel A of Figure 5 shows that the impact of DEX liquidity (X^*) exceeds that of CEX liquidity (S; the bid-ask spread) for buying informed traders, leading to a decline in β^*_{buy}. In contrast, β^*_{sell} shows the opposite reaction to σ. Moreover, since β^*_{sell} exhibits a stronger (positive) reaction to σ than the negative reaction of β^*_{buy}, the DEX involves more informed trading in expectation, i.e., $\beta^*_{buy} + \beta^*_{sell}$ increases.

Figure 5: Reaction of traders and liquidity to σ



Note: These figures are illustrated by using $z = 2.0$ and $\eta = 0.3$.

Numerical result 1. *When the asset becomes more volatile, informed sellers tend to cluster on the DEX and informed buyers on the CEX. The net effect is positive in the sense that the outflow of buyers is dominated by the inflow of sellers to the DEX.*

Intuitively, the convexity of the CFMM makes it less costly to add liquidity to the pool than to consume it. The asymmetric price impact means that an incentive to migrate (or stick) to the DEX is stronger for informed sellers. In other words, informed buyers exhibit a stronger reaction to a negative change in X: they are more eager to switch to the CEX. As a result, informed traders on the DEX exhibit an asymmetric reaction to a volatility shock. Namely, when selling (resp. buying) the asset, informed traders tend to cluster on the DEX (resp. the CEX).

Next, consider the behavior of liquidity traders. Panel B of Figure 5 shows that they tend to cluster on the CEX when the asset becomes more volatile. They compare the delay cost on the DEX ($\gamma\sigma$) to the expected trading cost on the CEX (S; the bid-ask spread). Since both of them are proportional to the asset volatility, σ has no direct impact on liquidity traders' venue choice. Instead, what matters is the *normalized* bid-ask spread, $\frac{S}{2\sigma}$, which captures the adverse selection problem for the CEX market maker that stems from traders' behavior.

In the above discussion, we have established that informed traders tend to gravitate toward the DEX in expectation (i.e., $\frac{\beta_{buy}+\beta_{sell}}{2}$ increases), which imposes more severe adverse selection on DEX market makers while mitigating that on the CEX market maker. It tightens the normalized bid-ask spread on the CEX and attracts liquidity traders to that exchange.

Numerical result 2. *When the asset becomes more volatile, liquidity traders tend to cluster on the CEX.*

Finally, Panels C and D of Figure 5 summarize the reaction of market liquidity to a change in asset volatility incorporating the above behavior of traders. Through their venue choice, traders have indirect effects and undermine the direct impact of σ on market liquidity, but they cannot offset or dominate the direct effect.

Proposition 7. *When the asset becomes more volatile, the DEX liquidity supply, X^*, declines.*[37]

Numerical result 3. *When the asset becomes more volatile, liquidity on the CEX, as measured by the bid-ask spread, deteriorates. The normalized bid-ask spread on the CEX, however, improves.*

The normalized bid-ask spread narrows because informed traders, in expectation, tend to cluster on the DEX, while liquidity traders are more likely to move to the CEX.

[37]This proposition is not limited to CPMMs and holds for CFMMs that satisfy Condition 1.

In contrast, the effective bid-ask spread on the CEX and the liquidity pools on the DEX positively correlate and deteriorate when the asset becomes more volatile.

5.4 Welfare

Finally, we provide a positive analysis regarding trader surplus in this economy. The zero-profit conditions of market makers imply that trading is a zero-sum game. As a result, a (negative) trading surplus arises due to the delay costs on the DEX (see Appendix C.8 for the welfare of each type of trader).

Proposition 8. *The expected ex-post trading surplus is proportional to the squared normalized bid-ask spread and is given by*

$$W = -\frac{1-\eta}{2}\left(\frac{S}{2\sigma}\right)^2 z\sigma. \tag{27}$$

Trading profits and costs cancel each other out due to the zero profit condition of market makers, i.e., a trade is just a transfer of money between liquidity traders and informed traders. The negative private utility from the delay costs drives W. It depends on $\alpha^2 = \left(\frac{S}{2\sigma}\right)^2$ because (i) the measure of liquidity traders on the DEX is proportional to α, and (ii) each incurs $\mathbb{E}[\gamma|\gamma < \alpha] = \frac{\alpha}{2}$ of delay costs in expectation.

Proposition 8 indicates that whether the advent of the DEX improves welfare depends on the modeling assumption of liquidity traders. As mentioned in Subsection 3.1, we can introduce negative values for γ representing traders' aversion toward the CEX due to, for example, cyber-security risks. With $\gamma < 0$, adding the DEX improves the aggregate welfare, as a portion of liquidity traders can avoid the risk by trading on the DEX.

Although the sign of the DEX's welfare impact depends on the modeling assumption, its magnitude can be measured through the bid-ask spread on the CEX, S. This is because the fraction of liquidity traders using the DEX and incurring the welfare cost (or enjoying the benefit) is proportional to S. An increase in the bid-ask spread on the CEX upon the introduction of the DEX suggests a large welfare impact.

6 Discussion

6.1 Empirical Implications

Novel empirical implications follow from our model. As exogenous variations, we can consider, for example, changes in asset volatility σ (or the degree of adverse selection) or ERC-20 tokens cross-listed on some CEXs and DEXs.[38]

Firstly, the equilibrium prices of the asset on the CEX are affected by an automated market.

Conjecture 1. *With the addition of a DEX with the CFMM, the midpoint of the bid and ask prices on the CEX tends to be higher than the expected value of the asset.*

The first conjecture is a natural consequence of the convex pricing of the CFMM and $\beta_{buy} < \beta_{sell}$, theoretically attested to by Corollary 1. A large body of literature examines asymmetric bid and ask prices, such as Ho and Stoll (1981) and Stoll (1989). In terms of the bid-ask spread that stems from adverse selection, studies have highlighted the asymmetry due to microstructure constraints, such as the discrete tick size (Anshuman and Kalay, 1998), and the asymmetric distribution of the value of assets (Bossaerts and Hillion, 1991). Our model proposes a new market structure that brings about the asymmetric prices in traditional limit-order markets and suggests that the midpoint tends to over-value the market expectation of the asset's value.

Moreover, the informativeness of the order flow tends to be asymmetric between exchanges.

[38] Uniswap started trading the ETH/WBTC pair on December 2020. WBTC is an ERC-20 token that is pegged to Bitcoin. Thus, the listing of WBTC on Uniswap can be seen as the advent of a DEX for the Ethereum and Bitcoin pair, which is previously traded on centralized exchanges.

Conjecture 2. *All else being equal, a higher asset volatility (or the degree of adverse selection) increases the informativeness of the order flow on the DEX and makes the order flow on the CEX less informative.*

This conjecture is based on Subsection 5.2. A higher degree of adverse selection makes it costly to trade on both venues. Informed traders tend to cluster on the same side of the market on the DEX, bearing a larger price impact, compared to liquidity traders, whose behavior is random. Thus, the order flow on the DEX tends to be information driven, while that on the CEX tends to be private-value driven.

Related to the above conjecture, buy and sell orders may react in different manners even if the magnitude of a trigger event is the same. The intuition follows from the combination of the facts that buying the asset bears a higher cost than the return from selling it (Corollary 3) and that informed traders tend to be more reactive than liquidity traders (Proposition 2). These facts imply that the informativeness of an order flow is asymmetric between sell and buy orders.

Conjecture 3. *All else being equal, when the asset becomes more volatile (or the degree of adverse selection), sell orders on the DEX are more likely to be followed by a negative innovation in returns than buy orders to be followed by a positive innovation. The opposite is true on the CEX.*

Moreover, the above prediction regarding the informativeness of the order flow has direct implications for market liquidity.

Conjecture 4. *All else being equal, an increase in the asset volatility (or the degree of adverse selection) is associated with a decline in the amount of the asset locked in the DEX, a wider effective bid-ask spread, and a narrower normalized bid-ask spread on the CEX.*

For example, our model predicts that the correlation between the effective bid-ask spread for the ETH/BTC pair on centralized exchanges (e.g., Coinbase) and its return volatility will be stronger after Uniswap starts trading the ETH/WBTC pair compared to the pre-Uniswap environment.

6.2 Arbitrage Trading and Liquidity Withdrawal

When information about $\tilde{v}$ becomes public at the end of $t = 1$, the marginal price on the DEX can differ from $\tilde{v}$. This deviation induces (unmodeled) arbitragers to trade until the arbitrage opportunity disappears. However, since market makers understand these arbitragers' behavior, as well as the realized value of Δz, they know that staking liquidity causes impermanent loss. At the end of $t = 1$, it is optimal for market makers to withdraw their liquidity since there is no Ethereum gas fees for liquidity removal in the model.[39]

Conditional on this behavior at the end of $t = 1$, the ex-ante expected return for liquidity providers is given by π_M^D in (25). Due to the liquidity withdrawal, only passive liquidity providers stay on the DEX at the end of $t = 1$, triggering $X \to x_{passive}$. After $t = 1$, the price on the DEX converges to $\tilde{v}$ due to a trade between passive liquidity providers and arbitragers, both of whom are unmodeled and exogenous in our model. This setting ensures our environment of the one-shot trading game.

6.3 Limitations of the Model

Fees. Introducing fees does not change our main discussions. On the DEX, liquidity takers pay trade execution fees, c, to liquidity providers. Moreover, all transactions on the DEX must be on the blockchain, meaning that they incur Ethereum gas fees, g. For example, informed traders on the DEX pay $p(\beta_{buy}) + g + c$ when buying the asset and obtain $p(-\beta_{sell}) - g - c$ when selling it. Conditional on liquidity removal at the end of $t = 1$, each liquidity provider obtains $wc\left(\eta\frac{\beta_{buy}+\beta_{sell}}{2} + z(1-\eta)\alpha\right) - g$ in expectation on top of π_M^D in (25) due to the fees. The results are quantitatively different from the main model, but the qualitative results stay the same.

[39] In Lehar and Parlour (2021), liquidity providers are assumed to stay in the pools after noise trading to earn transaction fees from arbitrage trading. Capponi and Jia (2021) consider liquidity providers who withdraw liquidity to avoid the impermanent loss with Ethereum gas fees.

However, these exogenous fees lead to several modeling issues. Firstly, the Ethereum gas, g, is paid in ETH, and its price may endogenously change in the equilibrium. Also, arbitragers may take profit opportunities only if the return is larger than fees, $c + g$. Investigating their behavior after the noise trading is cumbersome, as noise trading causes random shifts in liquidity pools and thus the profit for arbitragers. Liquidity providers must incorporate this choice of arbitragers and compare the return from the trading fee and impermanent loss with the Ethereum gas for liquidity withdrawal. Furthermore, introducing heterogeneous fee environments to each exchange makes the comparison between platforms arbitrarily biased. Incorporating all these points would be an interesting extension but beyond the scope of our analyses.

Information revelation. One of the limitations of our model is that it does not accommodate strategic informed traders with a long information horizon. When traders act on long-lived private information, we need to incorporate the public nature of data on blockchain. As mentioned in Appendix A, trading intentions on the DEX are stored in the mempool and wait for validation by blockchain miners. In most cases, the state of the mempool is publicly disseminated and observable for miners and traders. As suggested by Malinova and Park (2017), Daian et al. (2019), and Park (2021), a trader may extract other traders' private information by observing the mempool, generating front-running risk.

In our model, each informed trader is small and does not incorporate information revelation by the blockchain mempool. Such long-run behaviors and a strategic aspect of informed trading (e.g., Kyle, 1985) must be embedded in future research.

Endogenous delay. We cut corners in our analyses of delays on the DEX by assuming that a liquidity trader incurs a linear delay cost per transaction. This can be thought of as a situation where the mass of liquidity trading is sufficiently small compared to the block capacity so that all trades are settled with a constant (and deterministic) delay, i.e., the block time.

In general, however, a trader can shorten the expected waiting time by paying a higher transaction fee to blockchain miners. Since a miner processes better-paying transactions first, proposing a higher fee can move a trader forward in the queue. For example, Huberman et al. (2021) formulate the expected delay cost (the sum of the waiting time and fee payment) as an increasing function of the measure of traders waiting for verification.

Endogenizing the delay cost will certainly add new implications to our model. At the same time, however, we believe that the endogenous delay cost strengthens our results. If the delay cost is an increasing function of the measure of liquidity traders on the DEX (α), as suggested by Huberman et al. (2021), liquidity traders will be discouraged from participating in this exchange, leading to an even larger outflow to the CEX. Thus, an endogenous delay cost may work as an additional driving force to mitigate adverse selection for CEX market makers and improve CEX market liquidity.

7 Conclusion

This paper studies the equilibrium impact of adopting a decentralized exchange (DEX) with a novel market-making algorithm called a constant function market maker (CFMM). In the real financial market, DEXs with CFMMs and traditional centralized exchanges (CEXs) with limit-order mechanisms interact with each other. We construct a model to describe such a coexistence where traders are endogenously differentiated between the DEX and the CEX depending on their trading motives, i.e., informed or uninformed trading.

The model first shows that the amount of liquidity locked in the DEX has a positive impact on CEX liquidity (the bid-ask spread). We also characterize the profit function of market makers on the DEX who supply the asset and cash following the CFMM algorithm. Based on the derived profit function, we endogenize the amount of liquidity on the DEX and investigate how the DEX and CEX liquidity jointly react to an exogenous shock. The model proposes novel empirical implications that rely on the environment with the coexisting platforms.

In our model, we focus on a one-shot trading environment and abstract away from long-lived private information and sequential trading. When the information horizon becomes longer, informed traders must incorporate the speed of information revelation via their orders (as in Kyle, 1985). Moreover, price discovery in the long run is one of the two pillars that determine trader welfare. Thus, constructing a long-run model based on the current analyses is a topic for future research.

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Appendix

A Blockchain Technology

The blockchain can be seen as a novel way of managing and tracking transactions information. In the traditional world, we typically maintain a ledger that records participants' state information in a centralized manner, e.g., a bank acts as an intermediary. Bilateral transactions with no intermediation by a credible third party incur asymmetric information and settlement risk.

In contrast, on the blockchain platform, a ledger is not held by a particular entity, but is distributed across all participants in the network, called record keepers or blockchain miners. The distributed ledger system requires information about blockchain users to be a consensus among all record keepers. This highlights its first difference from traditional transactions, in which only a centralized authority keeps track of information. Due to its distributed nature, the blockchain is robust to a single point of failure and does not incur costs of building credibility.

A transaction with a distributed record-keeping system by blockchain goes as follows. Suppose that Alice wants to buy a cup of coffee at Bob's cafe by paying Bitcoin. Information about this transaction must be validated by blockchain miners. More precisely, the transaction is added to a block by a miner. A sequence of blocks are encrypted and become a blockchain. In the Bitcoin blockchain, for example, each miner in the network maintains a temporary list of unconfirmed transactions, called a *mempool*. Transactions in the mempool are yet to be recorded on the blockchain, and information on the mempool is public to the network. A miner picks one of the transactions in the pool and tries to validate it by executing costly computation following a certain algorithm. The fastest miner who solves the problem adds transaction information to a block (i.e., she mines a block). The reward for mining a block is a fee: when Alice initiates a transaction, she attaches a fee to her transaction, and the validating miner obtains the attached fee.[40]

In general, it is extremely difficult for one miner in the network to overturn the consensus. In the case of Bitcoin or Ethereum, for example, they leverage their computing power to solve a time-consuming cryptographic problem. This process is called proof of work (PoW), and the miner who performs it fastest is entitled to add a new block a chain.[41] Of course there can be multiple chains of blocks, because each miner can choose to which blockchain she adds a newly mined block. Following Nakamoto (2008), however, the longest chain is regarded as a valid chain. Therefore, if a malicious agent attempts to add fraudulent information to the transaction history (e.g., a double-spending problem), she must outpace all miners in the network and secretly generate a longer chain than other chains, which requires prohibitively high computing power. That is, information on the blockchain is (almost) free from tampering.

Moreover, Ethereum allows users to add complex scripts to the blockchain which describe the conditions under which transaction is verified and recorded. It implies that a transaction takes place only if the conditions in the code are fulfilled, and it is done automatically without any centralized third-party agencies. This type of automated contracts are called a *smart contract* following Szabo (1997).

[40] A miner also obtains a block reward, which is a constant amount of Bitcoin (or other cryptocurrency in other blockchains), when she mines a block. Although the block reward incentivizes miners to leverage their computing power, the amount of reward periodically shrinks and converges to zero in the future.

[41] There are several ways to reach a consensus, and different blockchains (including ETH 2.0) adopt different processes. For example, Saleh (2021) analyzes the viability of the proof of stake (PoS).

B Contingent Platform Choice

In this appendix, we check the robustness of our results by relaxing the assumption regarding liquidity traders' venue choice. We allow liquidity traders to choose their trading venue contingent on the realized sign of a private-value shock. In the following argument, we assume that the liquidity traders can choose their venue at $t = 0$ upon learning her trading size (i.e., buying or selling the asset). Due to the convexity of the CPMM pricing, we focus on the equilibrium in which the fractions of buying and selling liquidity traders on the DEX are asymmetric and given by $\alpha_{buy} \in (0, 1)$ and $\alpha_{sell} \in (0, 1)$, respectively.

By applying the same logic as the previous sections, informed traders' indifference conditions are given by

$$1 + a(\beta_{buy}, \alpha_{buy}) = p(\beta_{buy}), \tag{28}$$

$$1 - b(\beta_{sell}, \alpha_{sell}) = p(-\beta_{sell}), \tag{29}$$

where p is given by equation (9), and the ask and the bid prices are given by (18) and (19) with asymmetric α. As a result, the equilibrium measure of informed buyers and sellers can be expressed by reusing the previous equations.

Corollary 2. *Given $\alpha \equiv (\alpha_{buy}, \alpha_{sell})$, the equilibrium measures of informed buyers and sellers on the DEX, $(\beta^*_{buy}, \beta^*_{sell})$, solve the indifference conditions in (28) and (29). There exist a unique set of solutions and they are stable. β^*_i is increasing in X, σ, and α_i for $i \in \{buy, sell\}$.*

Thus, the reaction of informed traders in the partial equilibrium stays the same as the previous case with symmetric α in Proposition 1.

Now, consider the venue choice for liquidity traders. When a liquidity trader buys (resp. sells) the asset on the CEX, her trading cost (resp. reward) is the ask (resp. bid) price. In contrast, she pays or obtains the following symmetric price on the DEX:

$$P_{noise}(\alpha_{buy}, \alpha_{sell}) = \mathbb{E}_{(z_{buy}, z_{sell})}\left[P(\alpha_{buy} z_{buy} - \alpha_{sell} z_{sell})\right],$$

where $\mathbb{E}_{(z_{buy}, z_{sell})}$ is the expectation regarding the random variables z_i. As an example, assuming $z_i \sim U[0, z]$ and the CPMM generates the following explicit formula:

$$\begin{aligned} P_{noise}(\alpha_{buy}, \alpha_{sell}) &= \mathbb{E}_{(z_{buy}, z_{sell})}\left[\frac{X}{X - (\alpha_{buy} z_{buy} - \alpha_{sell} z_{sell})}\right] \\ &= \frac{X}{z^2 \alpha_{buy} \alpha_{sell}} \log \frac{(X + \alpha_{sell} z)^{X + \alpha_{sell} z}(X - \alpha_{buy} z)^{X - \alpha_{buy} z}}{X^X (X - z\Delta\alpha)^{X - z\Delta\alpha}}. \end{aligned} \tag{30}$$

When α is symmetric, the net expected amount of liquidity trading is zero, as z_{buy} and $-z_{sell}$ are symmetrically distributed, and buy and sell orders are netted out. In contrast, the asymmetric behavior of buy and sell liquidity traders prevents the orders from completely offsetting each other.

When deciding on the trading venue, a liquidity trader with delay cost γ compares the trading cost on the DEX (the LHS) and the CEX (the RHS):

$$\gamma\sigma \gtrless \begin{cases} 1 + a(\beta_{buy}, \alpha_{buy}) - P_{noise}(\alpha_{buy}, \alpha_{sell}) & \text{if a "buy" liquidity shock hits,} \\ P_{noise}(\alpha_{buy}, \alpha_{sell}) - (1 - b(\beta_{sell}, \alpha_{sell})) & \text{if a "sell" liquidity shock hits.} \end{cases}$$

Since γ uniformly distributes over $[0, 1]$, we obtain the following:

Corollary 3. *Given $(\beta_{buy}, \beta_{sell})$, the equilibrium measures of liquidity buyers and sellers on the DEX are given by the solution of the following equations.*

$$\alpha_{buy} = \frac{1 + a(\beta_{buy}, \alpha_{buy}) - P_{noise}(\alpha_{buy}, \alpha_{sell})}{\sigma},$$

$$\alpha_{sell} = \frac{P_{noise}(\alpha_{buy}, \alpha_{sell}) - (1 - b(\beta_{sell}, \alpha_{sell}))}{\sigma}.$$

For $i \in \{buy, sell\}$, α_i is decreasing in β_i and increasing in α_j for $j \neq i$.

The above result shows that the reaction of α_i in the partial equilibrium is the same as the previous analyses. The additional result brought by the asymmetric α is the strategic complementarity between liquidity buyers and sellers. Namely, liquidity buyers are more willing to trade on the DEX when more liquidity sellers participate in the DEX, and vice versa. This is because a larger trading volume on the opposite side of the market offsets the buy liquidity orders, leading to a smaller shift in the liquidity pools and a weaker price impact. Therefore, liquidity begets liquidity on the DEX, as in the traditional limit order markets (e.g., Pagano, 1989).

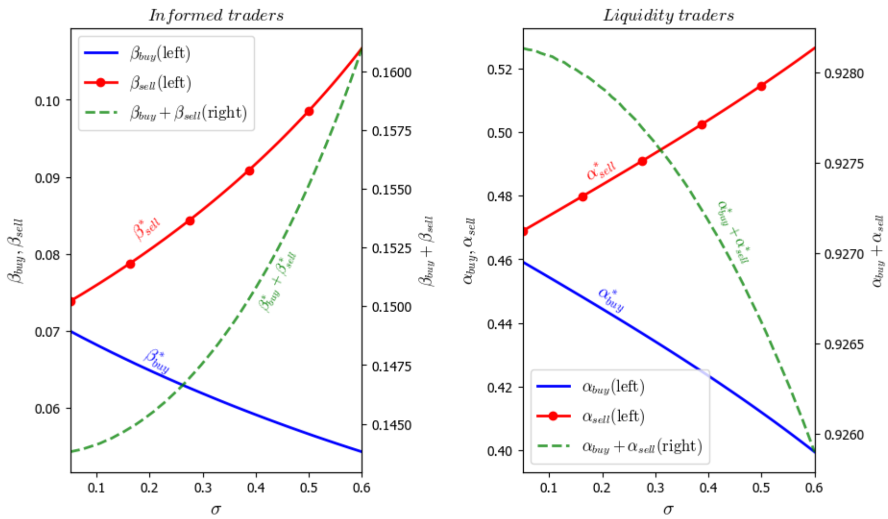
Finally, the expected profits for a market maker on the DEX is given by

$$\pi_M^D(X) = \frac{w\eta}{2}\left[\left(P(\beta^*_{buy}) - (1+\sigma)\right)\beta^*_{buy} + \left((1-\sigma) - P(-\beta^*_{sell})\right)\beta^*_{sell}\right] \tag{31}$$

$$+ w(1-\eta)\mathbb{E}[P_{noise}(\alpha_{buy}, \alpha_{sell})(\alpha_{buy}z_{buy} - \alpha_{sell}z_{sell})] \tag{32}$$

Once again, it is easy to check that $\pi^D_{M,IT} < 0$ and $\pi^D_{M,LT} > 0$, meaning that a market maker loses from informed trading and gains from liquidity trading. A larger mass of informed trading on the DEX, as well as a higher volatility of the asset, reduces DEX market makers' profits by worsening adverse selection.

Figure 6: Reaction of informed and liquidity traders



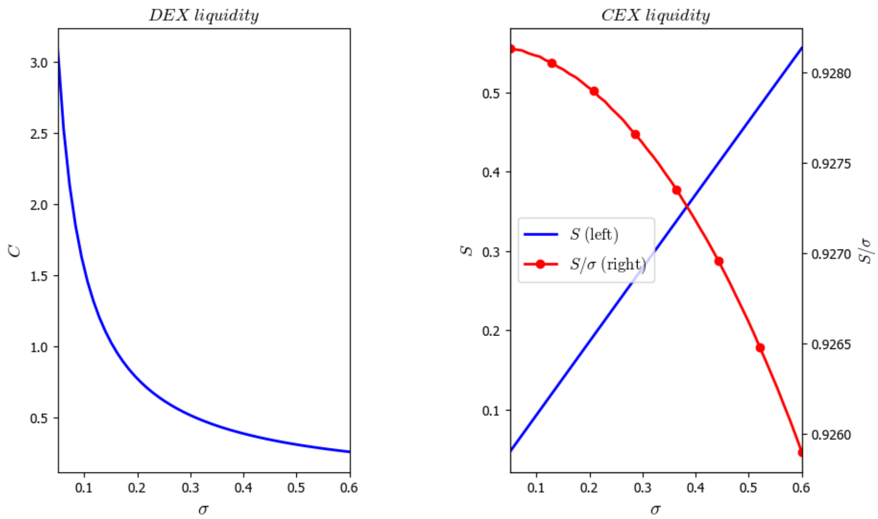
Note: This figures are illustrated by using $z = 2.0$ and $\eta = 0.3$. They are robust to other parameter values, as long as it holds that $z > z^*$.

Numerical result. In what follows, we take the CPMM as an example to see the robustness. Figure 6 plots the reaction of informed traders (the left panel) and liquidity traders (the right panel) on the DEX to an increase in the volatility of the asset. The asymmetric reaction of informed buyers and sellers on the left panel shows that the result in the previous analyses is robust to a change in the assumption on liquidity traders' venue choice. The right panel, however, shows that allowing a contingent venue choice adds a new implication regarding liquidity traders' behavior on the DEX.

Numerical result 4: *When the asset becomes more volatile, liquidity buyers tend to cluster on the CEX, while liquidity sellers tend to cluster on the DEX. The net effect is negative, i.e., outflow of liquidity traders from the DEX dominates inflow to the DEX.*

The net behavior of liquidity traders $\alpha_{buy} + \alpha_{sell}$ is different from that of informed traders. Intuitively, a liquidity trader on the DEX is not directly affected by the convexity of the CPMM algorithm *per se*, as she is uncertain about the aggregate trading volume (given by [30]). Thus, the asymmetric reaction of liquidity traders is driven by the asymmetric reaction of informed traders, that is, β_{buy} and β_{sell}. Since the expected mass of informed traders increases on the DEX, the bid-ask spread on the CEX shrinks which, in turn, induces liquidity traders to participate more on the CEX in expectation. Therefore, $\alpha_{buy} + \alpha_{sell}$ declines with σ.

Figure 7: Reaction of market liquidity



Note: This figures are illustrated by using $z = 2.0$ and $\eta = 0.3$. They are robust to other parameter values, as long as it holds that $z > z^*$.

Given the venue choice by traders, Figure 7 shows the reactions of market liquidity. The left panel shows the comparative static of DEX liquidity, measured by X^*, while the right panel illustrates the bid-ask spread (S) and the normalized bid-ask spread (S/σ). Since the *net* behavior of liquidity traders stays the same as the previous sections, so does the impact of the asset volatility on market liquidity.

Internet Appendix

C Proofs

In addition to the regularity conditions in Subsection 3.3, we assume the following technical condition to guarantee the uniqueness of the equilibrium and rule out abnormal behavior of prices. All conditions lead to the pricing function in Lemma 1 which provides intuition for the regularity conditions.

Condition 2 (Technical conditions)**.** *The CFMM function* $f : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ *with initial liquidity pool* (C, X) *satisfies the following:*
(v) $\frac{f_x(h(X-\delta;C,X),X-\delta)}{f_c(h(X-\delta;C,X),X-\delta)}$ *is decreasing in* X *if and only if* $\delta > 0$*, convex in* δ*, and differentiable with respect to* δ *and* X*. Moreover,* $\partial \frac{f_x(h(X-\delta;C,X),X-\delta)}{f_c(h(X-\delta;C,X),X-\delta)} / \partial X$ *is decreasing in* δ *;*
(vi) $a(\delta, X) \equiv |\frac{f_x(h(X-\delta;C,X),X-\delta)}{f_c(h(X-\delta;C,X),X-\delta)} - 1|$ *is log-submodular in* (X, δ)*.*

C.1 Proof of Proposition 1

First, let us show that $\alpha^*(\beta_{buy}, \beta_{sell}, X)$ uniquely exists. Equation (20) is

$$\alpha = \frac{S(\beta_{buy}, \beta_{sell}, \alpha)/\sigma}{2} = \frac{1}{2}\left[\frac{(1-\beta_{buy})\eta}{(1-\beta_{buy})\eta + z(1-\alpha)(1-\eta)} + \frac{(1-\beta_{sell})\eta}{(1-\beta_{sell})\eta + z(1-\alpha)(1-\eta)}\right]. \tag{33}$$

It holds that $S(\beta_{buy}, \beta_{sell}, 1) = 2\sigma$. Thus, the above equation has $\alpha = 1$ as a solution. Also, from the indifference conditions for informed traders, $0 < \beta_i < 1$ for all $\alpha \in [0, 1]$. Now, observe that (33) is equivalent to:

$$1-\alpha = \frac{z(1-\alpha)(1-\eta)}{2}\left[\frac{1}{(1-\beta_{buy})\eta + z(1-\alpha)(1-\eta)} + \frac{1}{(1-\beta_{sell})\eta + z(1-\alpha)(1-\eta)}\right].$$

When $\alpha \neq 1$,

$$1 = \frac{z(1-\eta)}{2}\left[\frac{1}{(1-\beta_{buy})\eta + z(1-\alpha)(1-\eta)} + \frac{1}{(1-\beta_{sell})\eta + z(1-\alpha)(1-\eta)}\right] \quad (34)$$

holds. Let the right hand side of this equation be $g(\alpha)$. Then, $g(0) < 1$. Moreover,

$$g'(\alpha) = \frac{z(1-\eta)}{2}\left[\frac{z(1-\eta)}{((1-\beta_{buy})\eta + z(1-\alpha)(1-\eta))^2} + \frac{z(1-\eta)}{((1-\beta_{sell})\eta + z(1-\alpha)(1-\eta))^2}\right] > 0$$

holds. Hence, an interior solution α^* exists if and only if $g(1) > 1$, which is equivalent to $z > \tilde{z}$ with

$$\tilde{z} = \frac{2\eta}{1-\eta}\frac{(1-\beta_{buy})(1-\beta_{sell})}{2-\beta_{buy}-\beta_{sell}}.$$

For any $\beta_{buy}, \beta_{sell}, \tilde{z} < z^* = \frac{\eta}{1-\eta}$ holds. Hence, Assumption 1 implies $g(1) > 1$. This also shows that α^* is unique if it exists. The negative impact of β_i on α^* in the partial equilibrium is straightforward.

Next, let us show that $\beta_i^*(\alpha, X)$ uniquely exists. Observe that

$$\frac{\{1+\sigma-p(\beta_{buy})\}(1-\beta_{buy})\eta}{\{p(\beta_{buy})-1\}(1-\eta)z} = 1-\alpha$$

holds. Since LHS is decreasing in β_{buy} and can take any positive value, there is an unique solution $\beta_{buy}^*(\alpha, X)$. Moreover, this implies that $\beta_{buy}^*(\alpha, X)$ is increasing in α. Following a similar argument, we can prove that $\beta_{sell}^*(\alpha, X)$ uniquely exists and is increasing in α. Furthermore, observe that

$$\frac{\{1+\sigma-p(\beta)\}(1-\beta)\eta}{\{p(\beta)-1\}(1-\eta)z} < \frac{\{p(-\beta)-(1-\sigma)\}(1-\beta)\eta}{\{1-p(-\beta)\}(1-\eta)z}$$

holds for $\beta > 0$. Hence, $\beta_{buy}^*(\alpha, X) < \beta_{sell}^*(\alpha, X)$ must hold.

C.2 Proof of Proposition 2

Equation (34) implies:

$$0 = \frac{\frac{\partial \beta^*_{buy}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X} z(1-\eta)}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X} z(1-\eta)}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2}$$

holds. By the conditions derived in Proposition 1, observe that:

$$\begin{aligned}
&-(1+\sigma)\eta\frac{\partial \beta^*_{buy}}{\partial X} - z(1-\eta)\frac{\partial \alpha^*}{\partial X} \\
&= (p_X(\beta^*_{buy}, X) + p_\beta(\beta^*_{buy}, X)\frac{\partial \beta^*_{buy}}{\partial X})((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)) \\
&- p(\beta^*_{buy}, X)\left(\eta\frac{\partial \beta^*_{buy}}{\partial X} + z(1-\eta)\frac{\partial \alpha^*}{\partial X}\right)
\end{aligned}$$

and this is equivalent to:

$$\begin{aligned}
&((1+\sigma - p(\beta^*_{buy}, X))\eta + p_\beta(\beta^*_{buy}, X)((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)))\frac{\partial \beta^*_{buy}}{\partial X} \\
&+ p_X(\beta^*_{buy}, X)((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)) \\
&= (p(\beta^*_{buy}, X) - 1)z(1-\eta)\frac{\partial \alpha^*}{\partial X}
\end{aligned}$$

Similarly, observe that:

$$\begin{aligned}
&-(1-\sigma)\eta\frac{\partial \beta^*_{sell}}{\partial X} - z(1-\eta)\frac{\partial \alpha^*}{\partial X} \\
&= (p_X(-\beta^*_{sell}, X) - p_\beta(-\beta^*_{sell}, X)\frac{\partial \beta^*_{sell}}{\partial X})((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)) \\
&- p(-\beta^*_{sell}, X)\left(\eta\frac{\partial \beta^*_{sell}}{\partial X} + z(1-\eta)\frac{\partial \alpha^*}{\partial X}\right)
\end{aligned}$$

and this is equivalent to:

$$
\begin{aligned}
&((p(-\beta^*_{sell}, X) - (1-\sigma))\eta + p_\beta(-\beta^*_{sell}, X)((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)))\frac{\partial \beta^*_{sell}}{\partial X} \\
&- p_X(-\beta^*_{sell}, X)((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)) \\
&= (1 - p(-\beta^*_{sell}, X))z(1-\eta)\frac{\partial \alpha^*}{\partial X}
\end{aligned}
$$

Combining these results, we obtain $\frac{\partial \alpha^*}{\partial X} < 0$ since p_X is negative (positive) when its first argument is positive (negative). Furthermore, at lease one of $\frac{\partial \beta^*_i}{\partial C_2}$ is positive.

We may further rewrite the above equations as:

$$
\begin{aligned}
A\frac{\partial \beta^*_{buy}}{\partial X} &= z(1-\eta)\frac{\partial \alpha^*}{\partial X} - \frac{(1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)}{p(\beta^*_{buy}, X) - 1} p_X(\beta^*_{buy}, X) \\
B\frac{\partial \beta^*_{sell}}{\partial X} &= z(1-\eta)\frac{\partial \alpha^*}{\partial X} + \frac{(1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)}{1 - p(-\beta^*_{sell}, X)} p_X(-\beta^*_{sell}, X)
\end{aligned}
$$

for some positive A, B. Lemma 1 Condition (vi) implies:

$$
\frac{p_X(\beta^*_{buy}, X)}{p(\beta^*_{buy}, X) - 1} < \frac{-p_X(-\beta^*_{sell}, X)}{1 - p(\beta^*_{sell}, X)} < 0
$$

Hence, we obtain:

$$
A\frac{\partial \beta^*_{buy}}{\partial X} > B\frac{\partial \beta^*_{sell}}{\partial X}
$$

Since at lease one of $\frac{\partial \beta^*_i}{\partial C_2}$ is positive, $\frac{\partial \beta^*_{buy}}{\partial X}$ is positive. If $\frac{\partial \beta^*_{sell}}{\partial X} \geq 0$, we are done. Suppose $\frac{\partial \beta^*_{sell}}{\partial X} < 0$. Then,

$$
\begin{aligned}
0 &= \frac{\frac{\partial \beta^*_{buy}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X}z(1-\eta)}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X}z(1-\eta)}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2} \\
&< \frac{\frac{\partial \beta^*_{buy}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X}z(1-\eta)}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}}{\partial X}\eta + \frac{\partial \alpha^*}{\partial X}z(1-\eta)}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2}
\end{aligned}
$$

holds. Hence, $\frac{\partial \beta^*_{buy}}{\partial X} + \frac{\partial \beta^*_{sell}}{\partial X} > 0$ since $\frac{\partial \alpha^*}{\partial X} < 0$.

C.3 Proof of Proposition 3

Since $p(\beta^*_{buy}) < (1+\sigma)$ and $p(\beta^*_{sell}) > (1-\sigma)$, we get $P(\beta^*_{buy}) < (1+\sigma)$ and $P(\beta^*_{sell}) > (1-\sigma)$. Therefore, $\pi^D_{IT} < 0$ holds. Next, observe that:

$$\frac{\partial \pi^D_{IT}}{\partial \beta_{buy}} = \frac{w}{2}\left(p(\beta_{buy}, X) - (1+\sigma)\right) < 0$$

because we are focusing on (X,α, β_{buy}, β_{sell}) that satisfy the equilibrium conditions, in particular, $p(\beta_{buy}, X) - (1+\sigma) < 0$. Similarly, we can show that π^D_{IT} is decreasing in β_{sell}.

C.4 Proof of Proposition 4

Since Q is symmetric, we may rewrite the profit function as:

$$\pi^D_{LT} = w \int_0^{\bar{z}} (P(\alpha\Delta z) - P(-\alpha\Delta z))\alpha\Delta z dQ(\Delta z).$$

This is positive because p is increasing in δ. Let us consider the comparative statics with respect to α. Observe that

$$(P(\alpha\Delta z) - P(-\alpha\Delta z))\alpha\Delta z = \int_0^{\alpha\Delta z} \{p(\delta) - p(-\delta)\}\, d\delta$$

Since p is increasing in δ, this object is increasing in α. Hence, π^D_{LT} is increasing in α. Furthermore, since p is decreasing in X if and only if $\delta > 0$, this object is decreasing in X. Combined with the fact that w is decreasing in X, π^D_{LT} is decreasing in X.

C.5 Proof of Proposition 5

First, we provide the formal statement of point (iii) in Proposition 5 that deals with endogenous variables in a global range.

Proposition 9 (Generalized point (iii) of Proposition 5). *Let $\bar{\beta}_i(X) \in \mathbb{R}_{++}$ be the solutions to $p(\bar{\beta}_{buy}, X) = 1 + \sigma$ and $p(-\bar{\beta}_{sell}, X) = 1 - \sigma$. $X^*(\alpha, \beta_{buy}, \beta_{sell})$ is decreasing in β_{buy} at $(\alpha, \beta_{buy}, \beta_{sell}) = (A, B_{buy}, B_{sell})$ if $\beta_{buy} \in [0, \bar{\beta}_{buy}(X^*(A, B_{buy}, B_{sell})))$. $X^*(\alpha, \beta_{buy}, \beta_{sell})$ is decreasing in β_{sell} at $(\alpha, \beta_{buy}, \beta_{sell}) = (A, B_{buy}, B_{sell})$ if $\beta_{sell} \in [0, \bar{\beta}_{sell}(X^*(A, B_{buy}, B_{sell})))$. Since equilibrium conditions require $p(\beta_{buy}) < 1 + \sigma$ and $p(-\beta_{sell}) > 1 - \sigma$, our interest lies in β_i that takes values within the range that guarantees point (iii) of Proposition 5.*

First, since $\left(P(\beta_{buy}) - (1+\sigma)\right)\beta_{buy} + \left((1-\sigma) - P(-\beta_{sell})\right)\beta_{sell}$, $\mathbb{E}[P(\alpha\Delta z)\alpha\Delta z]$ and w are decreasing in X, π_M^D is also decreasing in X for $\pi_M^D \geq 0$ given β_i and α. Next, for $\pi_M^D = 0$ to hold, we need:

$$\frac{\eta}{2}\left[\left(P(\beta_{buy}, X) - (1+\sigma)\right)\beta_{buy} + \left((1-\sigma) - P(-\beta_{sell}, X)\right)\beta_{sell}\right] + (1-\eta)\mathbb{E}[P(\alpha\Delta z)\alpha\Delta z] = 0.$$

Note that we take β_i and α as fixed and not functions of X. Since the LHS is decreasing in X and negative for sufficiently large X, there is a unique and stable solution X^* if the LHS is positive at $X = x_{passive}$. When the LHS is not positive at $X = x_{passive}$, it is negative for all $X > x_{passive}$.

As we have shown in Proposition 4, π_{LT}^D is increasing in α. Hence, X^* is increasing in α. Now, observe that

$$\frac{2}{w}\frac{\partial \pi_{IT}^D}{\partial \beta_{buy}} = p(\beta_{buy}) - (1+\sigma).$$

Since we have $\beta_{buy} \in [0, \bar{\beta}_{buy}(X^*(A, B_{buy}, B_{sell})))$, we get $p(B_{buy}, X) < (1+\sigma)$ for all $X \geq X^*(A, B_{buy}, B_{sell})$. Therefore, $\frac{\partial \pi_{IT}^D}{\partial \beta_{buy}^*} < 0$ holds for all $X \geq X^*(A, B_{buy}, B_{sell})$. Hence,

X^* is decreasing in β_{buy} at $(\alpha, \beta_{buy}, \beta_{sell}) = (A, B_{buy}, B_{sell})$ if $\beta_{buy} \in [0, \bar{\beta}_{buy}(X^*(A, B_{buy}, B_{sell})))$.. We can also show that X^* is decreasing in β_{sell} following a similar discussion.

C.6 Proof of Proposition 6

Let $S : \mathbb{R} \to \mathbb{R}^3$ be

$$S(X) = (\alpha^*(X), \beta^*_{buy}(X), \beta^*_{sell}(X))$$

and let $G : \mathbb{R}^3 \to \mathbb{R}$ be the function specified in equation (26). We will show that $G \circ S$ has a unique fixed point. By Propositions 1 and 5, we know that functions S and G are well-defined. Moreover, these functions are continuous and thus $G \circ S$ is continuous. Since $\lim_{X\to\infty} S(X) = (0, 1, 1)$ holds, $\lim_{X\to\infty} G \circ S(X) = \max\{x_{passive}, \tilde{X}\}$ holds where $\tilde{X}$ satisfies $p(1, \tilde{X}) - p(-1, \tilde{X}) = 2\sigma$. Also, note that $G \circ S(x_{passive}) \geq x_{passive}$ holds. Hence, $G \circ S$ has a fixed point in $[x_{passive}, \infty)$.

Next, we claim that $\tilde{\pi}^D_M(X; S(X)) = \frac{1}{w}\pi^D_M(X; S(X))$ is strictly decreasing in X. Note that $\pi^D_M(X^*; S(X^*)) = 0$ must hold if X^* is an interior solution for the DEX liquidity in equilibrium. Observe that:

$$\begin{aligned}
&\frac{d\tilde{\pi}^D_M(X; S(X))}{dX} \\
&= \frac{\eta}{2}\left[\left(p(\beta^*_{buy}(X), X) - (1+\sigma)\right)\frac{\partial \beta^*_{buy}(X)}{\partial X} + ((1-\sigma) - p(-\beta^*_{sell}(X), X))\frac{\partial \beta^*_{sell}(X)}{\partial X}\right] \\
&+ P_X(\beta^*_{buy}(X), X)\beta^*_{buy}(X) - P_X(-\beta^*_{sell}(X), X)\beta^*_{sell}(X) \\
&+ (1-\eta)\frac{\partial \mathbb{E}[P(\alpha^*(X)\Delta z; X)\alpha^*(X)\Delta z]}{\partial X}
\end{aligned}$$

Since $\alpha^*(X)$ is decreasing in X, the last term is negative by Proposition 4. Moreover, since $P_X(\beta^*_{buy}(X), X) < 0$ and $P_X(-\beta^*_{sell}(X), X) > 0$ hold, the second line is negative. Finally, we show that the first line is also negative. Equilibrium conditions lead to

$$p(\beta^*_{buy}(X), X) - (1+\sigma) = -\sigma \frac{(1-\eta)(1-\alpha^*)z}{(1-\beta^*_{buy}(X))\eta + (1-\eta)(1-\alpha^*)z},$$

$$(1-\sigma) - p(-\beta^*_{sell}(X), X) = -\sigma \frac{(1-\eta)(1-\alpha^*)z}{(1-\beta^*_{sell}(X))\eta + (1-\eta)(1-\alpha^*)z}.$$

If $\frac{\partial \beta^*_{sell}(X)}{\partial X} \geq 0$, these terms are negative, and the proof ends. Suppose that $\frac{\partial \beta^*_{sell}(X)}{\partial X} < 0$. We have shown that:

$$\frac{\frac{\partial \beta^*_{buy}(X)}{\partial X}}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}(X)}{\partial X}}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2} > 0.$$

By combining these results,

$$\begin{aligned}
&\left(p(\beta^*_{buy}(X), X) - (1+\sigma)\right) \frac{\partial \beta^*_{buy}(X)}{\partial X} + ((1-\sigma) - p(-\beta^*_{sell}(X), X)) \frac{\partial \beta^*_{sell}(X)}{\partial X} \\
&= -\sigma(1-\eta)(1-\alpha^*)z \\
&\times \left\{ \frac{\frac{\partial \beta^*_{buy}(X)}{\partial X}}{(1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)} + \frac{\frac{\partial \beta^*_{sell}(X)}{\partial X}}{(1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)} \right\} \\
&= -\sigma(1-\eta)(1-\alpha^*)z \left\{ (1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta) \right\} \\
&\times (\frac{\frac{\partial \beta^*_{buy}(X)}{\partial X}}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} \\
&+ \frac{\frac{\partial \beta^*_{sell}(X)}{\partial X}}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))}) \\
&< -\sigma(1-\eta)(1-\alpha^*)z \left\{ (1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta) \right\} \\
&\times \left\{ \frac{\frac{\partial \beta^*_{buy}(X)}{\partial X}}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}(X)}{\partial X}}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2} \right\} \\
&< 0
\end{aligned}$$

This shows that $\tilde{\pi}_M^D(X; S(X))$ is strictly decreasing in X. Hence, there is at most one interior equilibrium.

When $\tilde{\pi}_M^D(x_{passive}; S(x_{passive})) > 0$, $x_{passive}$ cannot be an equilibrium because providing additional liquidity is profitable, and the above argument shows that there is a unique interior equilibrium. When $\tilde{\pi}_M^D(x_{passive}; S(x_{passive})) \le 0$, there is no $X > x_{passive}$ such that $\tilde{\pi}_M^D(X; S(X)) = 0$. Thus, $x_{passive}$ is the equilibrium DEX liquidity.

The stability of the equilibrium directly follows from Propositions 2 and 5.

C.7 Proof of Proposition 7

From equation (34), we get (Condition A):

$$0 = \frac{\frac{\partial \beta^*_{buy}}{\partial \sigma}\eta + \frac{\partial \alpha^*}{\partial \sigma} z(1-\eta)}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}}{\partial \sigma}\eta + \frac{\partial \alpha^*}{\partial \sigma} z(1-\eta)}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2}.$$

From the indifference conditions, we may derive:

$$\begin{aligned} &(1-\beta^*_{buy})\eta - (1+\sigma)\eta\frac{\partial \beta^*_{buy}}{\partial \sigma} - z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma} \\ =& \left(p_\beta(\beta^*_{buy}, X^*)\frac{\partial \beta^*_{buy}}{\partial \sigma} + p_X(\beta^*_{buy}, X^*)\frac{\partial X^*}{\partial \sigma}\right)((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)) \\ &- p(\beta^*_{buy}, X^*)\left(\eta\frac{\partial \beta^*_{buy}}{\partial \sigma} + z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma}\right) \end{aligned}$$

Similarly,

$$\begin{aligned} &-(1-\beta^*_{sell})\eta - (1-\sigma)\eta\frac{\partial \beta^*_{sell}}{\partial \sigma} - z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma} \\ =& \left(-p_\beta(-\beta^*_{sell}, X^*)\frac{\partial \beta^*_{sell}}{\partial \sigma} + p_X(-\beta^*_{sell}, X^*)\frac{\partial X^*}{\partial \sigma}\right)((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)) \\ &- p(-\beta^*_{sell}, X^*)\left(\eta\frac{\partial \beta^*_{sell}}{\partial \sigma} + z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma}\right) \end{aligned}$$

These conditions imply (Condition B):

$$A\frac{\partial \beta^*_{buy}}{\partial \sigma} = -\frac{p_X(\beta^*_{buy}, X^*)\frac{\partial X^*}{\partial \sigma}}{p(\beta^*_{buy}, X^*) - 1}((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)) + \frac{(1-\beta^*_{buy})\eta}{p(\beta^*_{buy}, X^*) - 1} + z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma}$$

$$B\frac{\partial \beta^*_{sell}}{\partial \sigma} = \frac{p_X(-\beta^*_{sell}, X^*)\frac{\partial X^*}{\partial \sigma}}{1 - p(\beta^*_{sell}, X^*)}((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)) + \frac{(1-\beta^*_{sell})\eta}{1 - p(\beta^*_{sell}, X^*)} + z(1-\eta)\frac{\partial \alpha^*}{\partial \sigma}$$

Now, let us consider the break-even condition for the liquidity providers. Let $\tilde{\pi}^D_M(X; S(X)) = \frac{1}{w}\pi^D_M(X; S(X))$. Observe that:

$$\begin{aligned}
\frac{d\tilde{\pi}^D_M(X^*)}{d\sigma} &= \frac{\eta}{2}\left[\left(p(\beta^*_{buy}(X), X) - (1+\sigma)\right)\frac{\partial \beta^*_{buy}}{\partial \sigma} + ((1-\sigma) - p(-\beta^*_{sell}(X), X))\frac{\partial \beta^*_{sell}(X)}{\partial \sigma}\right] \\
&\quad - \frac{\eta}{2}(\beta^*_{buy} + \beta^*_{sell}) \\
&\quad + \frac{\eta}{2}\left\{P_X(\beta^*_{buy}, X^*)\beta^*_{buy}\frac{\partial X^*}{\partial \sigma} - P_X(-\beta^*_{sell}, X)\beta^*_{sell}\frac{\partial X^*}{\partial \sigma}\right\} \\
&\quad + (1-\eta)\frac{\partial \mathbb{E}[P(\alpha^*\Delta z; X)\alpha^*\Delta z]}{\partial \sigma}
\end{aligned}$$

Rearranging the terms,

$$\frac{d\tilde{\pi}^D_M(X^*)}{d\sigma} = -C_1\frac{\partial \beta^*_{buy}}{\partial \sigma} - C_2\frac{\partial \beta^*_{sell}}{\partial \sigma} - C_3\frac{\partial X^*}{\partial \sigma} + C_4\frac{\partial \alpha^*}{\partial \sigma} - C_5 = 0$$

must hold for $C_i > 0$ (Condition C). Suppose $\frac{\partial X^*}{\partial \sigma} \geq 0$. First, assume that $\frac{\partial \alpha}{\partial \sigma} \geq 0$. Then, by Condition B, both $\frac{\partial \beta^*_{buy}}{\partial \sigma}$ and $\frac{\partial \beta^*_{sell}}{\partial \sigma}$ must be positive. However, this is a contradiction to Condition A. Next, assume that $\frac{\partial \alpha^*}{\partial \sigma} < 0$. Then, Condition C implies that:

$$\left(p(\beta^*_{buy}(X), X) - (1+\sigma)\right)\frac{\partial \beta^*_{buy}}{\partial \sigma} + ((1-\sigma) - p(-\beta^*_{sell}(X), X))\frac{\partial \beta^*_{sell}(X)}{\partial \sigma} > 0.$$

Since $p(\beta^*_{buy}(X), X) - (1+\sigma) < 0$ and $(1-\sigma) - p(-\beta^*_{sell}(X), X) < 0$ hold, at least one of $\frac{\partial \beta^*_i(X)}{\partial \sigma}$ must be negative. Condition B implies that $A\frac{\partial \beta^*_{buy}}{\partial \sigma} \geq B\frac{\partial \beta^*_{sell}}{\partial \sigma}$ when $\frac{\partial X^*}{\partial \sigma} \geq 0$.

Hence, $\frac{\partial \beta^*_{sell}}{\partial \sigma} < 0$ must hold. Then, by Condition A, $\frac{\partial \beta^*_{buy}}{\partial \sigma} > 0$ holds. Now, following a similar argument as in the proof of Proposition 6,

$$
\begin{aligned}
&\left(p(\beta^*_{buy}, X) - (1+\sigma)\right) \frac{\partial \beta^*_{buy}}{\partial \sigma} + ((1-\sigma) - p(-\beta^*_{sell}(X), X)) \frac{\partial \beta^*_{sell}}{\partial \sigma} \\
&= -\sigma(1-\eta)(1-\alpha^*)z \\
&\times \left\{ \frac{\frac{\partial \beta^*_{buy}}{\partial \sigma}}{(1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)} + \frac{\frac{\partial \beta^*_{sell}}{\partial \sigma}}{(1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta)} \right\} \\
&= -\sigma(1-\eta)(1-\alpha^*)z \left\{(1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)\right\} \\
&\times (\frac{\frac{\partial \beta^*_{buy}}{\partial \sigma}}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} \\
&+ \frac{\frac{\partial \beta^*_{sell}}{\partial \sigma}}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))}) \\
&< -\sigma(1-\eta)(1-\alpha^*)z \left\{(1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta)\right\} \\
&\times \left\{ \frac{\frac{\partial \beta^*_{buy}}{\partial \sigma}}{((1-\beta^*_{buy})\eta + z(1-\alpha^*)(1-\eta))^2} + \frac{\frac{\partial \beta^*_{sell}}{\partial \sigma}}{((1-\beta^*_{sell})\eta + z(1-\alpha^*)(1-\eta))^2} \right\} \\
&< 0
\end{aligned}
$$

This is a contradiction. Therefore, it must be that $\frac{\partial X^*}{\partial \sigma} < 0$.

C.8 Proof of Proposition 8

Informed buyers and sellers on the DEX expect to obtain (in aggregate)

$$
W^D_{IT,i} = \begin{cases} \frac{\eta}{2}\beta_{buy}(1+\sigma - P(\beta_{buy}, X)) & \text{for } i = buy, \\ \frac{\eta}{2}\beta_{sell}(P(-\beta_{sell}, X) - 1 + \sigma) & \text{for } i = sell. \end{cases}
$$

Similarly, the *ex-post* aggregate utility for the liquidity traders on the DEX is

$$W_{LT,i}^{D} = \begin{cases} z_{buy}(1-\eta)\alpha(1-P(\alpha\Delta z, X) - \frac{\alpha}{2}\sigma) & \text{for } i = buy, \\ z_{sell}(1-\eta)\alpha(P(\alpha\Delta z, X) - 1 - \frac{\alpha}{2}\sigma) & \text{for } i = sell, \end{cases}$$

as they buy and sell z_{buy} and z_{sell} units in aggregate. Each market maker's expected trading profit from supplying w share of liquidity on the DEX is

$$V_{LP}(w; X) = w \times \begin{cases} -\mathbb{E}\left[(1-P(\alpha\Delta z, X))\alpha\Delta z\right] & \text{w.p. } 1-\eta \\ \sigma X - \beta_{buy}(1+\sigma - P(\beta_{buy}, X)) & \text{w.p. } \frac{\eta}{2} \\ -\sigma X + \beta_{sell}(P(-\beta_{sell}, X) - 1 + \sigma) & \text{w.p. } \frac{\eta}{2} \end{cases}$$

Obviously, the first line shows the liquidity trading with the private-value shock, and the second and the third ones are trading due to the common-value shock. Since the market makers supply X so that $\mathbb{E}[V_{LP}] = 0$, and a trade on the CEX is a zero-sum game, we obtain the result.

D Sequential Execution

In this appendix, we show that executing an order all at once (AAO) is the same as the sequential order execution.

Equivalence of post-trade liquidity pools. Suppose that there are n informed traders, and each of them has measure $w = \frac{1}{n}$ and places δ units of market buy order to the DEX (in the model, we assume $\delta = 1$). Note that the aggregate trading is of size δ. The initial state of the liquidity pool is denoted as (C_0, X_0) with $k \equiv C_0 X_0$. Note that the following discussion can be easily extended to the case with liquidity traders.

The first transaction is executed at price

$$p_1 = \frac{C_0}{X_0 - \delta w}$$

and the liquidity pool becomes

$$\begin{aligned} C_1 &= C_0 + p_1 \delta w = C_0 \frac{X_0}{X_0 - \delta w}, \\ X_1 &= X_0 - \delta w. \end{aligned}$$

By iterating, we obtain the following transition equations for the liquidity pools: for general $i = 1, 2, \cdots, n$,

$$\begin{aligned} C_i &= C_{i-1} \frac{X_{i-1}}{X_{i-1} - \delta w}, \\ X_i &= X_{i-1} - \delta w. \end{aligned}$$

The above equations imply that, after all (n) transactions are completed, the liquidity pools have

$$\begin{aligned} X_n &= X_0 - n\delta w = X_0 - \delta, \\ C_n &= C_0 \frac{X_0}{X_0 - n\delta w} = C_0 \frac{X_0}{X_0 - \delta}. \end{aligned}$$

Thus, the post-trade state of the pools with sequential execution is the same as that of AAO execution. The above result also implies that the profits for the market makers on the DEX stay the same even if we consider sequential execution of orders.

Equivalence of the execution price. Next, consider the expected trading cost (i.e., the execution price) for an informed trader. We consider a continuum of traders with measure β (by setting $n \to \infty$ with $\delta = 1$ and $w = \beta/n$ in the above example) and assume that traders' orders are independently executed following a Poisson process. Suppose that $y \in [0, \beta)$ orders have been executed before an informed trader gets to execute her order. From the above discussion, her order faces the following liquidity pools.

$$C_y = C_0 \frac{X_0}{X_0 - y}, \; X_y = X_0 - y.$$

Since her order is infinitesimal, it is executed at price

$$p(y) = \frac{C_y}{X_y} = \frac{C_0 X_0}{(X_0 - y)^2}.$$

Due to the independent Poisson process, $y \sim U[0, \beta]$. Thus, the expected execution price is given by

$$p = \frac{1}{\beta} \int_0^\beta p(y) dy = \frac{C_0}{X_0 - \beta},$$

which is identical to the execution price of each order in the case with AAO trade execution.